

On vertical distributions of velocity and pressure in an open-channel flow with rough bed

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ABSTRACT: Three-dimensional turbulent flows are observable over rough bed composing of gravels and boulders. To clarify these flow structures, it is desirable to employ quasi-three-dimensional analysis, which take account of non-hydrostatic pressure distribution. It is proved that the non-hydrostatic quasi-three-dimensional analysis method (Q3D-FEBS: Quasi-3D model considering Flow Equations on Boundary Surfaces), which considers flow equations on the water surface and bottom, in addition to the depth-averaged flow equations, is able to explain properly flood flows in real rivers.

In this study, Q3D-FEBS was extended to be applicable for the reverse flow near the riverbed caused by complex river topography. Specifically, two vertical velocity distributions divided into upper and lower layers were assumed, and interfacial boundary conditions between them were introduced properly to solve the lower layer flow. Velocity distribution of each layer is assumed to follow a biquadratic function. The interfacial boundary between the two layers is determined by the position of the maximum Reynolds stress. We investigated the two-layer biquadratic velocity distribution and demonstrated its ability to explain reverse flow and the secondary flow of Prandtl's second kind in the flow with complex topography.

Then, a method of non-hydrostatic quasi-three-dimensional two-layer flow analysis (Q3D-FEBS-2L) was proposed.

1 INTRODUCTION

1.1 Development of non-hydrostatic quasi-3D model

There is a demand for quasi-3D models, which solve the non-hydrostatic pressure distributions in addition to the three-dimensional velocity distributions. Quasi-three-dimensional models have been developed to analyze three-dimensional flows associated with the complex topography of river channels and beds. Non-hydrostatic quasi-3D models have been applied especially in the field of coastal engineering (e.g., flow around offshore structures (Antuono et al. 2022)). There has also been a development in the field of river engineering.

Table 1 shows examples of our works using non-hydrostatic quasi-3D analysis method. Uchida et al. (2011) developed the General Bottom Velocity Computation (GBVC) method to analyze three-dimensional flood flows and riverbed variations, such as river confluence flow (Koshiishi et al. 2012) & meandering compound channel flow (Uchida et al. 2013). The GBVC method solves the equations of motion (depth-averaged, surface) and the vorticity equation (depth-averaged). They used the vertical velocity distributions in a cubic function associated with the boundary conditions (B.C. 1 through 4) in Table 2. The GBVC methods solves the depth-integrated vorticity equation to obtain the bottom surface velocity.

Furthermore, Uchida et al. (2014) developed the GBVC-DWL method (The General Bottom Velocity Computational method coupled with the Dynamic rough Wall Law), which can solve the flow near riverbed. The GBVC-DWL method solves the equations of motion for the near-bed layer (vortex layer) and the roughness layer, in addition to a set of equations of motion of the GBVC method. The GBVC-DWL method introduces non-equilibrium bottom boundary conditions that incorporates the interaction between fluid and roughness. This method has been applied in flood flows in real rivers (e.g., flow around a pier (Uchida et al. 2015)). The quasi-3D models

Table 1. Existing non-hydrostatic quasi-3D analysis method

Analysis method	Fundamental equations of motion	Analysis target	Characteristics
SBVC (Uchida et al. 2009)	Continuity equation (depth integrated)	River confluences (Uchida et al. 2009), etc.	Hydrostatic model, quadratic function
GBVC(-EWL) (Uchida et al. 2011)	- Equation of motion (water surface, depth integrated) - Vorticity equation (depth integrated)	Submerged groins (Uchida et al. 2011), River confluence (Koshiishi et al. 2012), compound channel flow (Uchida et al. 2013), etc.	Non-hydrostatic model, cubic function, equilibrium models for sediment-transport(EWL)
GBVC(3)-DWL (Uchida et al. 2014)	- Continuity equation (depth integrated, vortex layer, roughness layer) - Equation of motion (water surface, depth integrated, vortex layer, roughness layer)	compound channel flow (Uchida et al. 2014), Rough bed with submerged boulders (Uchida et al. 2014), around a pier (Uchida et al. 2015), etc.	Non-hydrostatic model, cubic function, Non-equilibrium models for sediment-transport(two-phase model)
GBVC4-DWL (Uchida et al. 2016)	Vorticity equation (depth integrated)	Rough bed with submerged boulders (Uchida et al. 2016), etc.	Non-hydrostatic model, cubic function, evaluate the vorticity production term by biquadratic function, Dynamic rough wall law (DWL)
Non-equilibrium models for sediment-transport (Takemura et al. 2020)	- Continuity equation (bed load, suspended load) - Equation of motion (bed load, suspended load)	Riverbed variation analysis method (Takemura et al. 2020, Hasebe et al. 2022)	bed load and suspended load integrated
Q3D-FEBS (Takemura et al. 2019)	- Continuity equation (depth integrated) - Equation of motion (water surface, bottom surface, depth integrated)	hydraulic jump experiments (Takemura et al. 2019), alluvial rivers(compound channel flow (Hasebe et al. 2022)), mountain rivers(Takemura et al. 2019), etc.	Non-hydrostatic model, cubic function, solve the equation of motion of bottom surface, coupling with Non-equilibrium models for sediment-transport
Q3D-FEBS-2L	- Continuity equation (depth integrated) - Equation of motion (water surface, bottom surface, upper layer integrated, lower layer integrated)	(Flow regime with a particularly complex cross-sectional profile with riverbed irregularities)	(Non-hydrostatic model) , biquadratic function of 2-layer

of the GBVC series are within the framework of 2D analysis methods, which reduces computation time compared to 3D analysis methods.

Takemura et al. (2019) developed Q3D-FEBS (Quasi-3D model considering Flow Equations on Boundary Surfaces), which solves the equation of motion on the bottom surface. Q3D-FEBS also uses vertical velocity distribution in a cubic function. Q3D-FEBS and the non-equilibrium sediment model (Takemura et al. 2020) have been combined to solve the flow field and sediment transport in an alluvial river (compound channel flow (Hasebe et al. 2022)). Takemura et al. (2019) applied Q3D-FEBS to mountain rivers with large rocks and gravels. However, there was a problem against the motion of large gravels in the complex flow field near the riverbed. Quasi-3D models that can carefully explain the velocity and pressure fields near the riverbed is required to overcome this problem.

1.2 Concept of the present research

This study proposes a quasi-3D flow model valid for rivers with complex cross-sections composing of large rocks and gravels. The vertical velocity distributions used in existing Q3D-FEBS are focused to provide better explanation on this issue. This study consists of two steps (Figure 1.). In Step 1, new vertical velocity distributions are proposed in order to properly describe the flow structures near the riverbed. In Step 2, proposed velocity distributions are incorporated into the framework of Q3D-FEBS.

Specifically, in Step 1, the two-layer biquadratic function distributions of velocity are proposed; The computational domain is divided into two layers, lower layer is influenced by bed roughness and the other is by water depth. Taking account of these velocity distributions and boundary conditions, possible shape of flow velocity and pressure distribution are examined by using time-averaged three-dimensional numerical data of LES analysis by Fukushima et al.

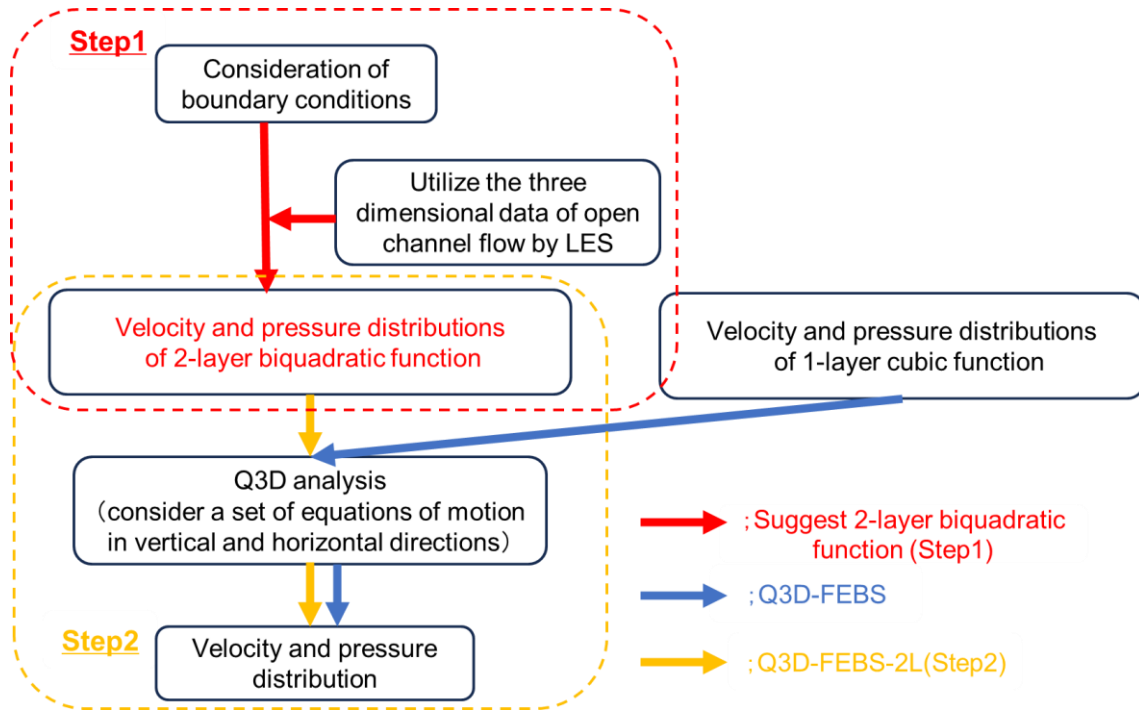


Figure 1. Flowchart of this research

(2022). In step 2, a non-hydrostatic quasi-3D two-layer flow analysis method (Q3D-FEBS-2L) that can solve 3D flow controlled by large roughness is proposed.

2 2-LAYER BIQUADRATIC VELOCITY FUNCTION

2.1 Problems of vertical velocity distribution expressed by a cubic function

The Q3D-FEBS method, which assume the vertical velocity distribution by a cubic function, has been used to solve 3-D flow in various rivers. The vertical velocity distribution in a cubic function is given by Eq. (1). Here, the coefficients A-D in Eq. (1) are determined by considering four boundary conditions (1) through (4) shown in Table 2. In Eq. (1), the unknowns are surface velocity u_s , depth-averaged velocity U_i , and bottom velocity u_b .

$$u_i(\eta) = A\eta^3 + B\eta^2 + C\eta + D \quad (1)$$

Where $u_1 = u, u_2 = v, \eta = (z_s - z)/h, A = -12U_i + 8u_s + 4u_b, B = 12U_i - 9u_s - 3u_b, C = 0, D = u_s$. The bottom surface is defined as the distance away by 3% of the water depth from the riverbed surface. The flow field below the bottom surface is assumed to be the non-computational domain.

Two problems arise in applying Eq. (1) to the flow around large rocks and gravels. First, the water depth around large gravels is highly variable due to the uncertainty of riverbed surface. It is difficult to calculate the velocity and pressure distributions and the movement of sediment particles behind large gravels.

In addition, boundary conditions for the vertical velocity distribution (Table 2.) indicate that while the velocity at the bottom surface is taken into account, the velocity gradient near the bottom surface is not. This suggests that reverse and separated flows are not adequately accounted for. Setting the boundary conditions near the riverbed would have a significant effect on the analysis of complex flows near the bed.

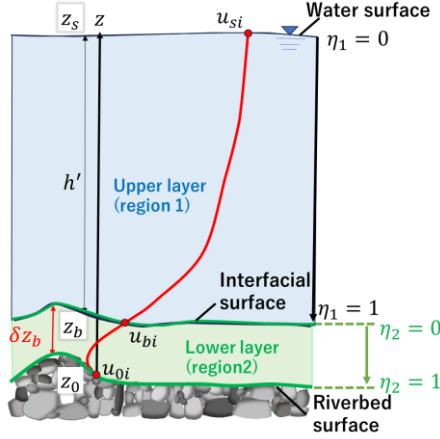


Figure 2. Concept of 2-layer biquadratic function distributions

Table 2. Boundary conditions of velocity distributions

upper layer($\eta_1 = \frac{z_s - z}{h'}$)	No.	lower layer($\eta_2 = \frac{z_b - z}{\delta z_b}$)	No.
$u_{1i}(0) = u_{si}$	1	$u_{2i}(0) = u_{bi}$	6
$u_{1i}(1) = u_{bi}$	2	$u_{2i}(1) = 0$	7
$\int_0^1 u_i(\eta_1) d\eta = U_{1i}$	3	$\int_0^1 u_i(\eta_2) d\eta_2 = U_{2i}$	8
$\left. \frac{\partial u_i(\eta_1)}{\partial \eta} \right _{\eta_1=0} = 0$	4	$\left. \frac{\partial u_i(\eta_2)}{\partial z} \right _{z=z_b} = \left. \frac{\partial u_i(\eta_1)}{\partial z} \right _{z=z_b}$	9
$\left. \frac{\partial^2 u_i(\eta_1)}{\partial \eta_1^2} \right _{\eta_1=1} = 0$	5	$\left. \frac{\partial^2 u_i(\eta_2)}{\partial \eta_2^2} \right _{\eta_2=0} = 0$	10

(About unknowns)
 U_{1i} : upper layer thickness mean velocity,
 U_{2i} : lower layer thickness mean velocity,
 ρ : density, h' : upper layer thickness,
 δz_b : lower layer thickness

2.2 Dividing the computational domain into the upper and lower layers by the maximum position of Reynolds stress

Nezu et al. (1993) defined the Wall Region in the range ($y/h < 0.15 \sim 0.2$), which is the layer strongly influenced by roughness. It has been demonstrated that the Wall Region has momentum and Reynolds stress distribution characteristics different from the Intermediate Region ($0.15 \sim 0.2 < y/h < 0.6$) and the Free-Surface Region ($0.6 < y/h$) (Nezu et al. 2001). This concept is applied to the layers significantly affected by roughness. In this study, the location of the maximum position of the Reynolds stress is defined as the interfacial surface between the two layers.

The Reynolds stress is modeled using the eddy viscosity and the first-order derivative term of the velocity. The location of the maximum Reynolds stress coincides with the location where the second-order derivative term of the flow velocity is zero. The position where the second derivative term is zero corresponds to the inflection point of the vertical velocity distribution. As a result, vertical velocity distribution can be divided into two regions with different characteristics at the maximum position of the Reynolds stress. In this study, the location of the interfacial surface is set to 20% of the water depth, which is setuped on the basis of the results of 3-D numerical analysis (Chapter 2.4).

2.3 The 2-layer biquadratic function distributions

The computational domain ranges from the riverbed to the water surface, and is divided into two layers based on the location of the maximum Reynolds stress. The lower layer is strongly affected by bed roughness, and the upper layer is located above it. Each layer is evaluated with a biquadratic function as in Eq. (2) and (3) (upper layer: coefficients $A_1 \sim E_1$, lower layer: coefficients $A_2 \sim E_2$). Each coefficient is derived by applying the boundary conditions indicated in Table 2. (upper layer: B.C. (1) through (5), lower layer: B.C. (6) through (10)). The unknowns of Eq. (2) and (3) are surface velocity u_s , upper layer averaged velocity U_{1i} , lower layer averaged velocity U_{2i} , and interfacial surface velocity u_b .

$$u_i(\eta_1) = A_1 \eta_1^4 + B_1 \eta_1^3 + C_1 \eta_1^2 + D_1 \eta_1 + E_1 \quad (2)$$

Where $A_1 = -(5/3)(3u_{bi} + 5u_{si} - 8U_{1i})$, $B_1 = (4/3)(9u_{bi} + 16u_{si} - 25U_{1i})$, $C_1 = -2(3u_{bi} + 7u_{si} - 10U_{1i})$, $D_1 = 0$, $E_1 = u_{si}$.

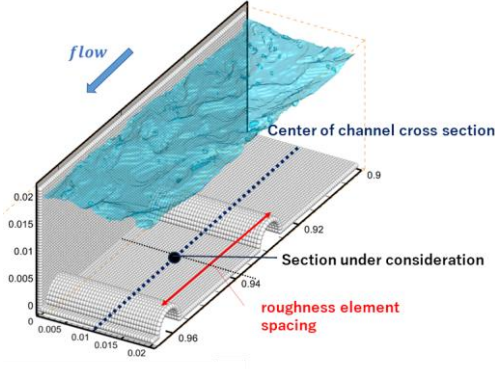


Figure 3. Numerical experiment channel

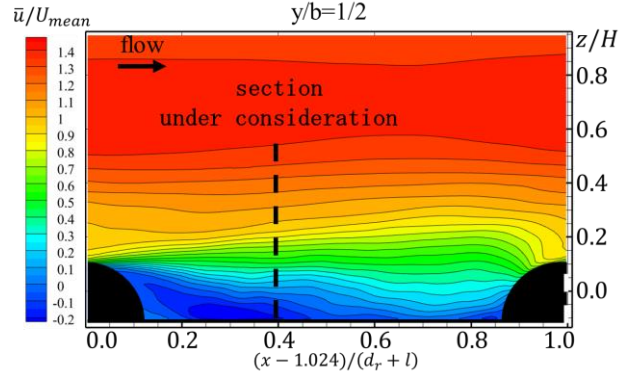


Figure 4. Main flow velocity contour in the center cross section of the channel

$$u_i(\eta_2) = A_2\eta_2^4 + B_2\eta_2^3 + C_2\eta_2^2 + D_2\eta_2 + E_2 \quad (3)$$

Where $A_2 = 5(3u_{bi} - 4U_{2i} + I)$, $B_2 = -2(8u_{bi} - 10U_{2i} + 3I)$, $C_2 = 0$, $D_2 = (\delta z_b/h')((20/3)(u_{si} - U_{1i}) - 4(u_{si} - u_{bi})) (= I)$, $E_2 = u_{bi}$.

The boundary conditions (1) through (4) (upper layer) are the same as the flow with cubic velocity function (Eq. (1)). Correspondingly, the B.C. (6) through (9) are imposed on the flow of the lower layer. The boundary condition of the velocity is zero (B.C. (7)) is given at the height of the riverbed surface ($\eta_2 = 1$) shown in Figure 2. The B.C. (9) is set in the common condition between the upper and lower layer. The B.C. (5) and (10), that the second-order derivative term of the velocity is zero are considered for each layer.

The pressure distribution of the two-layer flow follows that of the Q3D-FEBS. In Q3D-FEBS, the pressure distribution is obtained by depth integration of the vertical equation of motion along with velocity distributions, which are given by (u, v) (Eq. (2) and (3)) and (w) expressed by a linear distribution (Eq. (4)).

$$w(\eta_i) = W + \frac{\delta w}{2}(1 - 2\eta_i) \quad (4)$$

Where $\delta w = w_s - w_b$ ($i = 1$), w_b ($i = 2$). W is the depth-integrated vertical velocity derived from the continuity equation.

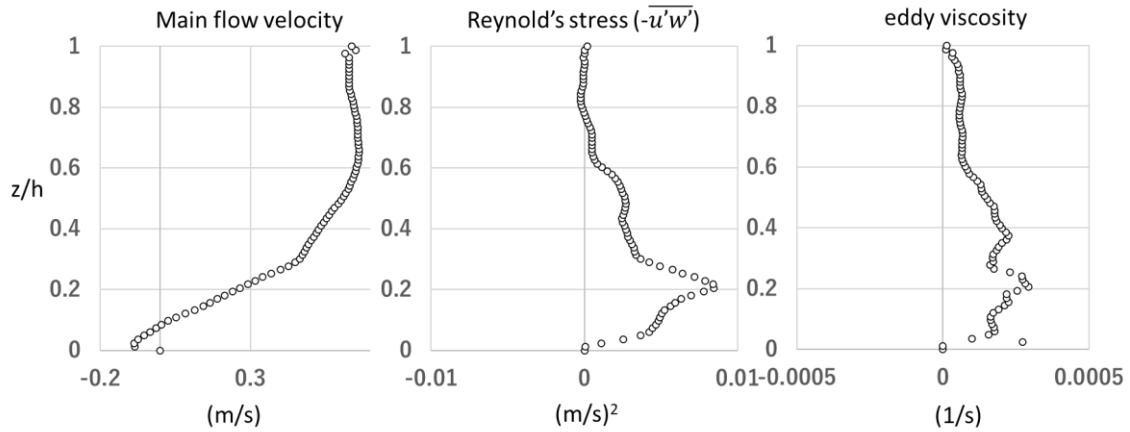


Figure 5. Interface position of the upper layer and lower layer

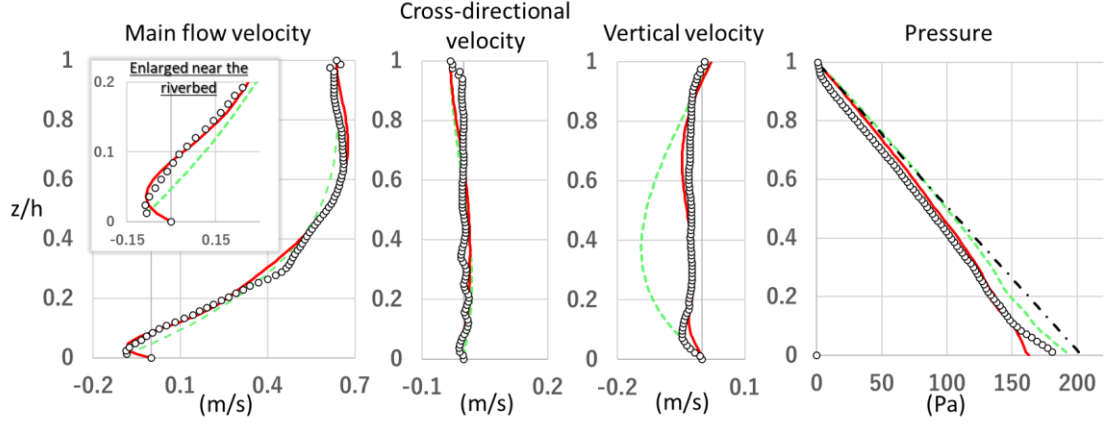


Figure 6. Results of velocity and pressure distributions
(Plot; 3D analysis, dotted line; cubic function distribution, solid line; two-layer distribution, chain line; hydrostatic pressure distribution)

2.4 3-D numerical analysis data and interfacial surface of the upper and lower layers

In Step 1, the possible shape of the two-layer biquadratic velocity functions (Eq. (2) and (3)) is derived, utilizing the results of the time-averaged velocity distribution data from the 3D numerical analysis of LES (Fukushima et al. 2022). For the 3D numerical analysis, semi-cylinders roughness elements are equally spaced in the open channel flow with a small aspect ratio ($b/h=1.41$) (Figure 3.). The analysis revealed that the flow field has velocity-dip in the main flow due to the secondary flow of Prandtl's second kind (Figure 4.)

The location of the interfacial surface was studied using the time-averaged 3D numerical data. Figure 5. shows vertical distributions of the main flow velocity, the Reynolds stress, and the eddy viscosity in the separation zone behind the semi-cylinder roughness elements at the center of channel cross section (Figure 3.). As shown in Figure 5, the height of the inflection point of the vertical velocity distribution generally corresponds to the height where the Reynolds stress and the eddy viscosity has maximum values. Similar investigation was conducted at different points along the center of channel cross section, and it was confirmed that the inflection point of the vertical velocity distribution and the maxima of the Reynolds stress and the eddy viscosity appears at the height of 15-25% of the water depth above the channel bed surface. Therefore, the height of the interfacial surface δz_b is set $0.2h$ in this study.

2.5 Velocity and pressure distributions

The vertical distributions of u , v , and w are shown in Figure 6. The dotted lines represent the cubic function distribution (Eq. (1)), while the solid lines represent distributions of the two layers (Eq. (2) and (3)). They are derived by using time-averaged data of the 3D numerical analysis. The reverse velocity of the main flow (u) is seen in Figure 6. This shows an enlarged view of main flow velocity (u) near the riverbed, where the two-layer model tends to represent the separated flow. The secondary flow velocity (v , w) is very small compared to the main flow velocity (u).

Similarly, Figure 6 shows the calculation result of the pressure distribution. In Step 1, the pressure distribution was derived by using the time-averaged velocities u , v , and w of 3D numerical analysis. The results show that calculated pressure distribution seem to differ from measured distribution especially near the riverbed. In order to get the realistic velocity and pressure distributions, it is necessary to examine them in the framework of Step 2. In Step 2, the equations of motion are solved using a non-hydrostatic quasi-3D two-layer flow analysis method (Q3D-FEBS-2L).

3 NON-HYDROSTATIC QUASI-3D TWO-LAYER FLOW ANALYSIS METHOD (Q3D-FEBS-2L)

In Step 2, non-hydrostatic quasi-3D two-layer flow analysis method (Q3D-FEBS-2L) is to be performed using two-layer biquadratic velocity function. The continuity equation in the upper and lower layer is expressed of Eq. (5). In addition to the upper layer (Eq. (6)) and lower layer (Eq. (7)), the equations of motion on the water surface (Eq. (8)) and on the interfacial surface (Eq. (9)) are solved. The Reynolds stress is evaluated by using the eddy viscosity shown in Eq. (10). The unknowns of Eq. (2) and (3) are determined by Eq. (5) through Eq. (9).

$$\frac{\partial h}{\partial t} + \frac{\partial(u_{1i}h' + u_{2i}\delta z_b)}{\partial x_i} = 0 \quad (5)$$

$$\frac{\partial U_{1i}h'}{\partial t} + \frac{\partial U_{1j}U_{1i}h'}{\partial x_j} = -gh' \frac{\partial z_s}{\partial x_i} - \frac{1}{\rho} \frac{\partial h' \overline{p_1'}}{\partial x_i} - \frac{p'_b}{\rho} \frac{\partial z_b}{\partial x_i} - \frac{\partial h' \overline{u'_{1i}u'_{1j}}}{\partial x_j} + \frac{1}{\rho} \frac{\partial h' \overline{\tau_{1ij}}}{\partial x_j} - \hat{S} \frac{\widehat{\tau_{bi}}}{\rho} \quad (6)$$

$$\begin{aligned} \frac{\partial U_{2i}\delta z_b}{\partial t} + \frac{\partial U_{2j}U_{2i}\delta z_b}{\partial x_j} = & -g\delta z_b \frac{\partial z_b}{\partial x_i} - \frac{1}{\rho} \frac{\partial \delta z_b \overline{p_2'}}{\partial x_i} + \frac{p'_b}{\rho} \frac{\partial z_b}{\partial x_i} - \frac{p'_0}{\rho} \frac{\partial z_0}{\partial x_i} \\ & - \frac{\partial \delta z_b \overline{u'_{2i}u'_{2j}}}{\partial x_j} + \frac{1}{\rho} \frac{\partial \delta z_b \overline{\tau_{2ij}}}{\partial x_j} - \hat{S} \frac{\hat{\tau}_{0i}}{\rho} \end{aligned} \quad (7)$$

$$\frac{\partial u_{si}}{\partial t} + u_{sj} \frac{\partial u_{si}}{\partial x_j} = -g \frac{\partial z_s}{\partial x_i} - \frac{1}{\rho h'} \frac{\partial z_s}{\partial x_i} \frac{\partial p'}{\partial \eta} \bigg|_s - \frac{\tilde{S}}{\rho h'} v_t \frac{\partial^2 u_i}{\partial \eta_1^2} \bigg|_s \quad (8)$$

$$\frac{\partial u_{bi}}{\partial t} + u_{bj} \frac{\partial u_{bi}}{\partial x_j} = -g \frac{\partial z_s}{\partial x_i} - \frac{1}{\rho} \frac{\partial p_b'}{\partial x_i} - \frac{1}{\rho \delta z_b} \frac{\partial z_b}{\partial x_i} \frac{\partial p'}{\partial \eta} \bigg|_b + \frac{1}{\rho} \frac{\partial \tau_{ij}}{\partial x_i} \bigg|_b - \frac{\hat{S}}{\rho \delta z_b} \frac{\partial \widehat{\tau_{bi}}}{\partial \eta} \bigg|_b \quad (9)$$

$$v_t = \frac{\kappa u_* h}{6} \quad (10)$$

Where $i, j = 1, 2$ (x, y direction), subscripts 1; upper layer, 2; lower layer, h' : upper layer thickness, δz_b : lower layer thickness, p' : non-hydrostatic pressure, $\overline{(u'_i u'_i)}$: momentum transport, τ_{ij} : shear force in direction i, j , τ_{zi} : shear force in direction z , κ : Karman constant (0.41), u_* : friction velocity, h : water depth, $\widehat{\cdot}$: interface variables, $\tilde{\cdot}$: water surface variables; S : correction coefficient due to the longitudinal and transverse gradients of the riverbed.

4 CONCLUSION

The objective of this study is to develop a numerical quasi-3D model valid for flows in rivers with complex topography. An attempt was made to improve the existing non-hydrostatic quasi-three-dimensional flow model (Q3D-FEBS). The main conclusions are as follows:

1. In order to calculate the flow with a strong influence of bed roughness, the computational domain is divided into two layers. The vertical velocity distributions of each layer are assumed to be a biquadratic function.
2. An interfacial surface between two-layers is considered as the location of the maximum Reynolds stress. Using the time-averaged data of the three-dimensional LES analysis for complex open channel-flow, the interfacial surface position is set at 20% of the water depth.
3. The biquadratic velocity distributions seem to represent the reverse flow and secondary flow by using the time-averaged 3D numerical results of complex open channel flow (Step1).

4. It is necessary to conduct the analysis of non-hydrostatic quasi-three-dimensional two-layer flow (Q3D-FEBS-2L) for further understanding of this issue (Step 2).

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