

Study of Water Storage and Drainage in the Embankments during Rainfall Using a Boundary-Fitted Hydrological Modeling based on Water Balance Analysis

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Abstract - This study introduces a boundary-fitted hydrological model to simulate rainfall-induced infiltration, storage, and drainage within river embankments. Motivated by increasing flood risks and embankment failures under extreme rainfall, the model integrates surface runoff and unsaturated subsurface flow within a water balance framework. It accounts for infiltration up to a maximum rate, surface runoff, and exfiltration when internal saturation exceeds capacity. Flow dynamics are represented using shallow water equations and Darcy's law, with vertical and horizontal components modeled through a tank-based structure fitted to embankment geometry. Validation against large-scale rainfall and ponding experiments by the Public Works Research Institute demonstrated accurate reproduction of saturation patterns and runoff volumes. Applications to full-scale railway embankments and the Kushiro River levee confirmed the model's ability to capture delayed infiltration under high-intensity rainfall and fine-grained soils. Key parameters— R_s , h_{ma} , and h_{mc} —were calibrated and analyzed for physical consistency, revealing significant influence of soil type and void ratio on water retention. The proposed model provides a practical tool for assessing embankment stability and flood resilience under varied rainfall scenarios. Future work will refine flow representation at saturated-unsaturated interfaces, incorporate soil structural changes during prolonged rainfall, and extend validation across diverse soil and climatic conditions.

Keywords: large embankment, rainfall, water balance analysis, storage, boundary-fitted model, drainage

1. Introduction

In recent years, record-breaking rainfall events have been observed almost annually, leading to an increasing number of embankment failures. Not only external water levels but also rainfall on embankments have intensified, causing slope failures even in gently sloped embankments. For embankments such as those along the Tone River, where the riverside slope reaches 70%, the increased surface area can result in greater rainfall infiltration, potentially causing collapse.

When simulating rainfall infiltration using conventional seepage analysis methods, the infiltration flux is typically applied to the rainfall boundary surface based on rainfall intensity. However, once the pressure head at the ground surface reaches zero, further infiltration ceases [1]. Rain infiltrating into the embankment is calculated using Richards' equation based on soil constants and soil-water characteristic curves, which determines the pressure head distribution and groundwater level. In practice, the actual amount of infiltrated rainfall remains uncertain, and the presence of air voids causes reduced infiltration during the initial rainfall stage. Reproducing this phenomenon requires complex two-phase flow analysis, which is a significant challenge [2].

Japanese embankments have been repeatedly raised and reinforced after disasters, and standardized compaction and construction methods have resulted in reliable data on surface soil structures. Therefore, if the amount of rainfall infiltration and storage within an embankment can be evaluated, it is possible to estimate the spatiotemporal distribution of water within the embankment during heavy rainfall with reasonable accuracy. However, direct measurement of rainfall storage is difficult, and analytical techniques remain complex, necessitating the development of new analytical approaches.

Based on the concept of watershed water balance, if rainfall on an embankment, outflow from the embankment, and temporal changes in saturation can be observed, integrated hydrological analysis combining surface and unsaturated flow

can be applied to estimate rainfall infiltration and water storage within the embankment [3] [4] [5]. This study focuses on the phenomenon where infiltration is hindered by air voids and suction. Using rainfall experiment data from large-scale model embankments at the PWRI (Public Works Research Institute) and RTRI(Railway Technical Research Institute), we improve the water balance analysis method for saturated and unsaturated flow, introduce new parameters, and verify them to examine infiltration and unsaturated characteristics. Finally, we discuss challenges in estimating moisture distribution within embankments based on water balance analysis.

2. Governing Equations for Saturated and Unsaturated Flow

2.1. Overview of Large-Scale Embankment Experiments and Unsaturated Characteristics by PWRI

The Public Works Research Institute conducted large-scale embankment experiments to evaluate reinforcement methods under prolonged high water levels and rainfall [6]. This study uses rainfall experiment data from untreated embankments composed of sandy soil (permeability coefficient: 3.65×10^{-5} m/s). Rainfall amount, outflow, and saturation distribution within the embankment were measured.

Figure 2 schematically illustrates the relationship between rainfall on the embankment, outflow, and water storage on the surface and inside the embankment. Equation (1) expresses the water balance during rainfall:

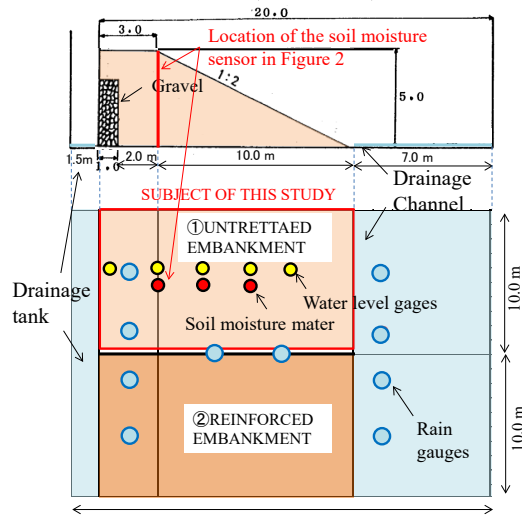


Fig. 1: Schematic of the large-scale embankment model used in rainfall experiments

$$\frac{dS_e}{dt} + \frac{dS_s}{dt} = \frac{1}{3.6 \times 10^6} R_a \cdot A - Q_{out1} - Q_{out2} - Q_{sur} \quad (1)$$

S_e : Water storage within the embankment (m^3), S_s : Water storage on the embankment surface (m^3), R_a : Rainfall intensity on the embankment (mm/h), A : Projected surface area of the embankment (m^2), Q_{out1} : Outflow from the gravel side (m^3/s), Q_{out2} : Outflow from the toe of the embankment (m^3/s), Q_{sur} : Surface runoff from the embankment (m^3/s)

Experimental results show that saturation does not increase immediately after rainfall begins; instead, saturation rises gradually from the surface downward, taking about 20 hours for saturation at 1.5 m depth to increase. This stepwise movement suggests that suction and air voids hinder downward water movement.

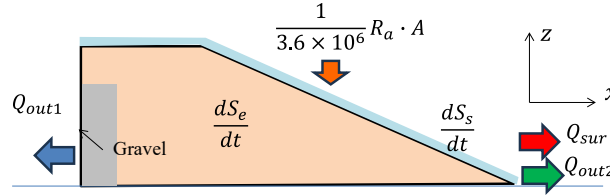


Fig. 2: Conceptual representation of water balance components during rainfall

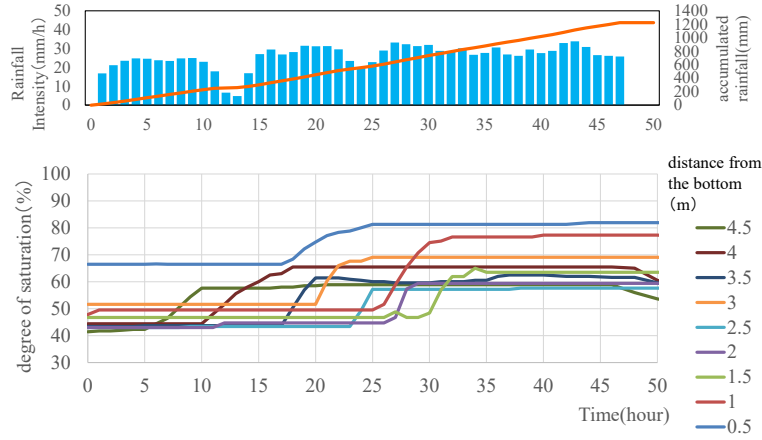


Fig.3: Rainfall intensity and depth-wise saturation variation measured by soil moisture sensors in untreated levee

2.2. Boundary-Fitted Hydrological Model Considering Unsaturated Characteristics

This study focuses on the phenomenon observed during rainfall experiments where infiltrated water moves stepwise within the embankment due to the influence of suction and entrapped air. To represent this behavior, the embankment is conceptualized as a series of interconnected tanks, each corresponding to a specific soil layer. The water content within each tank is expressed as a storage height h_t , which is equivalent to the volumetric water content of the soil.

At the initial stage of rainfall, water movement is minimal because suction and air pockets restrict infiltration. To incorporate this effect, the model assumes that when the storage height h_t is below a threshold h_{mc} (movement initiation storage height), water does not move. Once h_t exceeds h_{mc} , water begins to move horizontally; however, vertical movement remains restricted by air pockets until h_t surpasses another threshold h_{ma} (air pocket release storage height). Below h_{ma} , horizontal movement occurs, but vertical movement is negligible.

Rainfall infiltration at the embankment surface is also influenced by suction and air pockets. Therefore, when the surface tank's storage height h_t is less than h_{ma} , only a small portion of rainfall, denoted as R_s , infiltrates into the embankment, while the remainder becomes surface runoff. The parameter R_s represents the infiltration rate under conditions where air exchange occurs gradually near the surface, reflecting the slow interaction between water and air during early rainfall stages. When h_t exceeds h_{ma} , the entire rainfall amount infiltrates into the embankment.

To compute flow within the embankment, governing equations are formulated using a vertical coordinate transformation $\sigma = (z - z_0)/D$, enabling calculations along the embankment geometry [7]. The continuity and momentum equations (Equations 2–10) describe surface flow, infiltration, and internal water movement. For surface flow, the shallow water equations are applied, considering slope angle variations. Unsaturated hydraulic conductivity is expressed using the Mualem–Van Genuchten relationship, which accounts for effective saturation S_e and soil parameters [8]. When saturation occurs near the toe of the embankment, excess water that cannot be stored is treated as seepage outflow, calculated using Equation (10).

$$\frac{\partial h_s}{\partial t} + \frac{\partial U_s h_s}{\partial x} + \frac{1}{3.6 \times 10^6} (R_i - R_a) = 0 \quad (R_i = R_s \ (R_a \geq R_s), \quad R_i = R_a \ (R_a < R_s)) \quad (2)$$

$$\frac{\partial U_s h_s}{\partial t} + \frac{\partial U_s U_s h_s}{\partial x} + U_s \frac{1}{3.6 \times 10^6} (R_i - R_a) = -g h_s \frac{\partial z_s}{\partial x} - \frac{1}{\cos \theta_b} \frac{g n^2 U_s |U_s|}{h^{1/3}} \quad (3)$$

$$\frac{\partial h_t}{\partial t} + \frac{\partial U h_t}{\partial x} + \frac{1}{3.6 \times 10^6} R_i - \frac{w_{b\sigma}}{\cos \theta_b} = 0 \quad (R_i = R_s \ (R_a \geq R_s), \quad R_i = R_a \ (R_a < R_s)) \quad (4)$$

$$\frac{\partial h_t}{\partial t} + \frac{\partial U h_t}{\partial x} + \frac{w_{\sigma t}}{\cos \theta_t} - \frac{w_{\sigma b}}{\cos \theta_b} = 0 \quad (5)$$

$$\frac{\partial h_t}{\partial t} + \frac{\partial U h_t}{\partial x} + \frac{w_{\sigma t}}{\cos \theta_t} = 0 \quad (6)$$

$$U = 0 \quad (h_t < h_{mc}) \quad U = -k_{wi} \frac{\partial h_p}{\partial x} - k_{wi} \sigma \frac{\partial D}{\partial x} \quad (h_t \geq h_{mc}) \quad (7)$$

$$w = 0 \quad (h_t < h_{ma}) \quad w = -k_{wi} \frac{1}{D} \frac{\partial}{\partial \sigma} (\sigma D + h_p) \quad (h_t \geq h_{ma}) \quad (8)$$

$$k_{wi} = k_a \cdot S_e^\xi \left\{ 1 - \left(1 - S_e^{\frac{1}{m}} \right)^m \right\}^2 \quad (9)$$

$$R_i = -\frac{\partial}{\partial x} \int_0^1 U h_t d\sigma \quad (10)$$

h_s : Surface flow depth (m), U_s : Surface flow velocity (m/s), R_a : Rainfall intensity (mm/h), R_i : Infiltration/exfiltration rate into the embankment (mm/h), z_s : Surface water level (m), n : Manning's roughness coefficient ($m^{-1/3} \cdot s$), g : Gravitational acceleration (m/s^2), θ_b : Slope angle of the embankment

h_t : Tank storage height (m), h_p : Pressure head in the tank (m), U : Horizontal flow velocity within the embankment (m/s), w : Vertical flow velocity within the embankment (m/s), $w_{\sigma t}$, $w_{\sigma b}$: Vertical flow velocities at the top and bottom of the tank (m/s), θ_t , θ_b : Angles of the tank top and bottom surfaces, k_a : Permeability coefficient of the embankment (m/s), D : Embankment height at each location (m), k_{wi} : unsaturated hydraulic conductivity (m/s), h_{mc} , h_{ma} : threshold storage heights for movement initiation and air pocket release, R_s : infiltration parameter under air exchange conditions

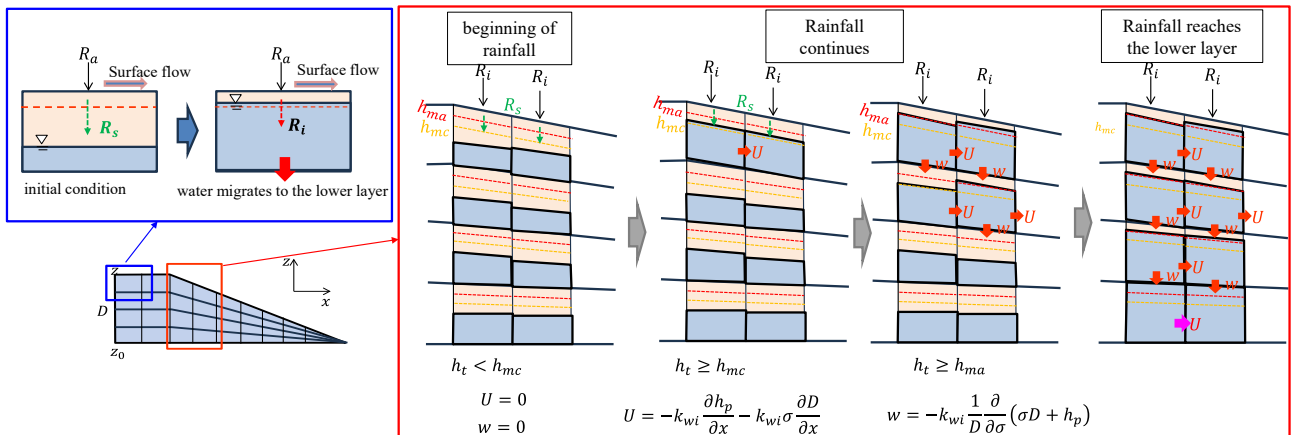


Fig. 3: Boundary-fitted hydrological model for saturated-unsaturated flow analysis

3. Hydraulic Analysis and Discussion Based on Large-Scale Model Experiments

3.1. Application to Rainfall Experiments at the PWRI

The water balance analysis described in Section 2 was applied to rainfall experiments conducted on large-scale model embankments by the Public Works Research Institute. The governing equations (Equations 1–10) were solved simultaneously, and the parameters h_{mc} , h_{ma} , and R_s were adjusted iteratively so that the calculated water distribution and outflow matched the experimental observations. Initial storage heights for each tank were set based on the measured saturation distribution at the start of the experiment, while other conditions were aligned with experimental specifications.

Figure 5 compares the observed and simulated saturation distributions at 6, 8, and 46 hours after rainfall began. Experimental data show that between 6 and 8 hours, saturation near the toe of the embankment increased to approximately 0.7, spreading toward the shoulder. The simulation reproduced this trend, with saturation levels of 0.6–0.7 near the toe. Furthermore, experimental results indicated minimal saturation change near the crest during early rainfall, reflecting delayed infiltration. The model successfully captured this phenomenon, showing gradual storage in the surface layer and delayed infiltration. At 46 hours, the simulated saturation contour for 0.8 closely matched the experimental wetting front, and the model reproduced the movement of the saturated zone from the toe toward the gravel side. Figure 6 illustrates the comparison of water balance components between measurements and simulations.

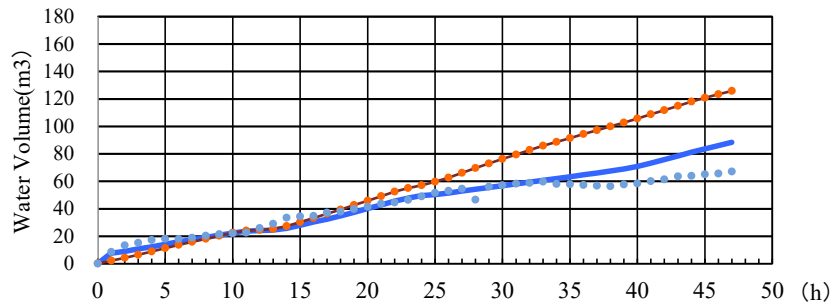


Fig. 5: Comparison of observed and simulated water balance components during PWRI rainfall experiment

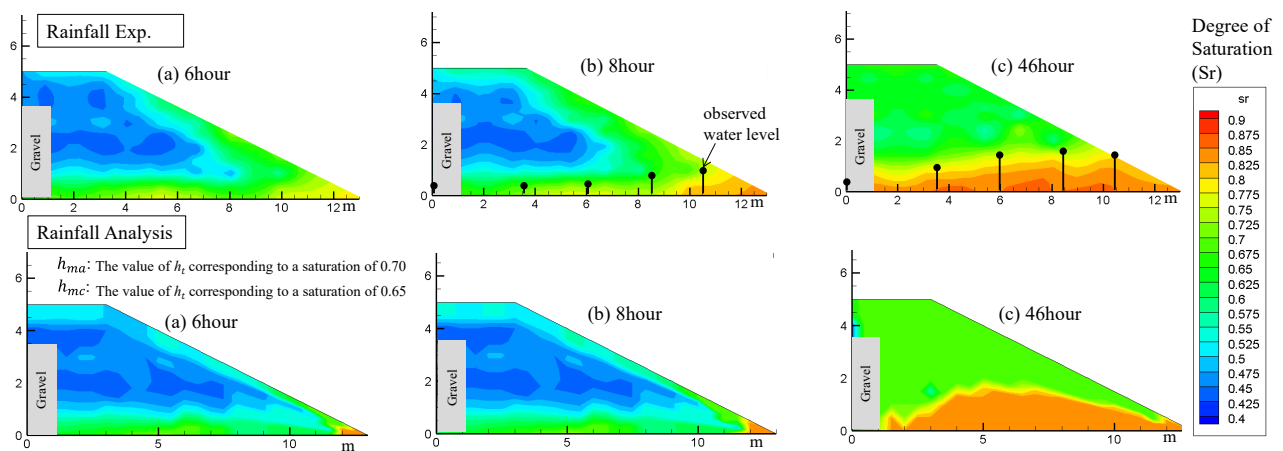


Fig. 6: Observed vs. simulated saturation distribution during PWRI rainfall experiment

3.2. Application to RTRI Rainfall Experiment (Sand Soil)

Sato et al. conducted full-scale rainfall experiments on railway embankments (12.0 m wide, 6.0 m deep, 5.0 m high, sandy soil with fine particles) under a rainfall intensity of 30 mm/h [9]. The proposed model was applied to these experiments, adjusting h_{mc} , h_{ma} , and R_s to match observed saturation distributions. Outflow was not measured in this experiment.

Figure 7 compares observed and simulated saturation distributions from 20 to 80 hours after rainfall began. Experimental data show that saturation near the crest increased during the first 40 hours, followed by downward migration. The simulation reproduced this trend, with saturation moving downward after 50 hours. However, after 70 hours, experimental data indicated high saturation near the toe, while the simulation showed little change in that region. This discrepancy suggests that horizontal water movement requires further refinement in the model.

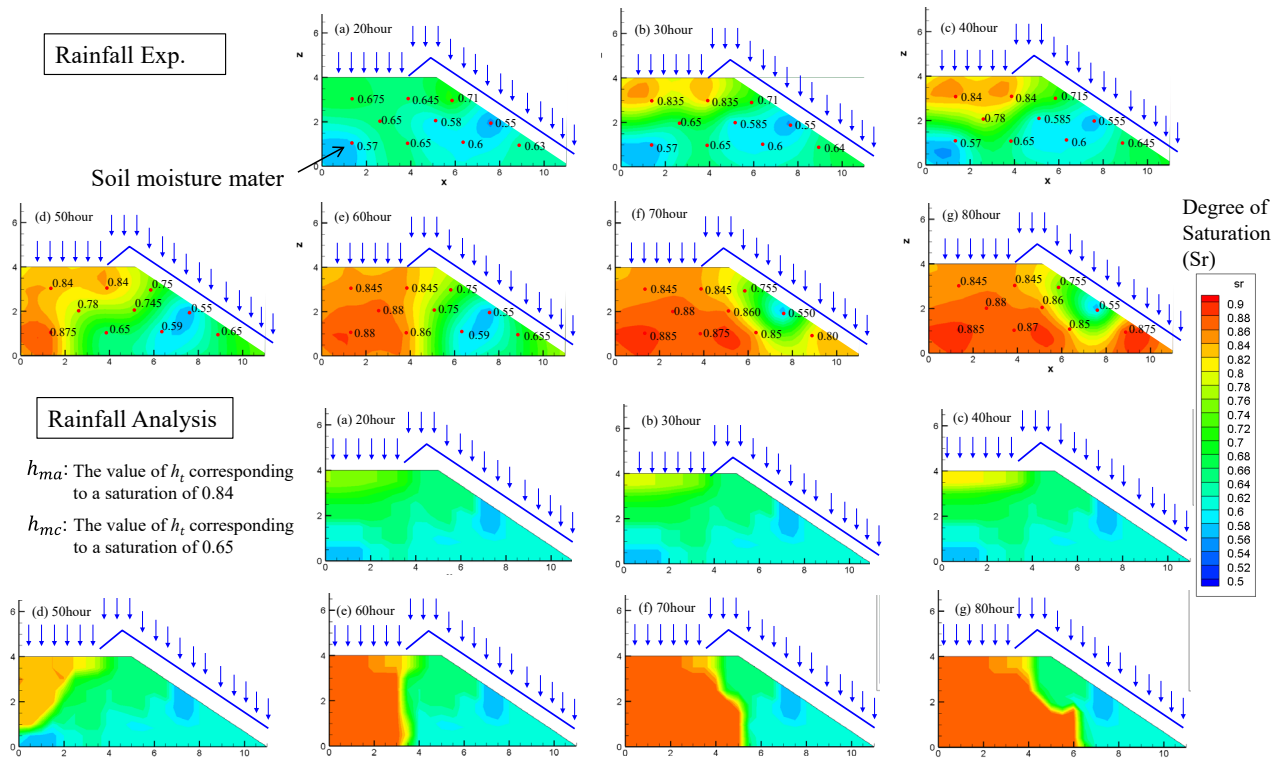


Fig. 7: Observed vs. simulated saturation distribution during RTRI railway embankment rainfall experiment

3.3.Application to Kushiro River Field Embankment (Silty Soil)

A test embankment was constructed along the Kushiro River to evaluate the effect of slope flattening for reinforcement [10]. The original sandy embankment was widened using silty soil with a slope ratio of 1:3. Rainfall experiments applied up to 20 mm/h for approximately 30 hours. Saturation was measured using soil moisture sensors installed in the widened section. The model was calibrated by adjusting h_{mc} , h_{ma} , and R_s to match observed saturation distributions.

Figure 8 compares observed and simulated saturation distributions at 24 and 30 hours after rainfall began. Experimental data show that saturation increased in the widened section during the first 24 hours, which the simulation successfully reproduced. After 30 hours, experimental saturation reached 0.9–0.95, and the simulation produced similar results.

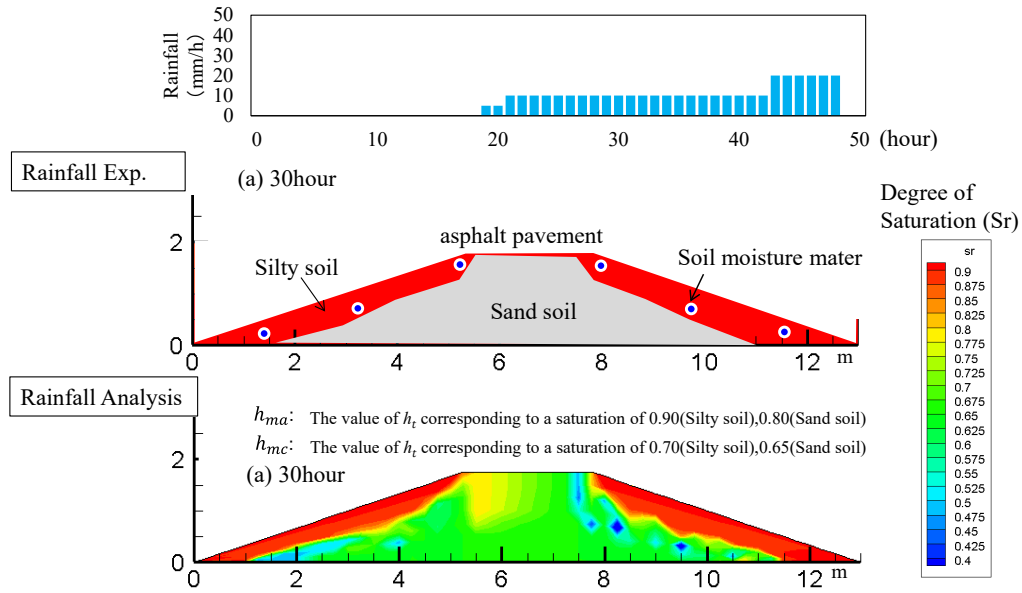


Fig. 8: Observed vs. simulated saturation distribution during Kushiro River embankment rainfall experiment

2.6. Physical Interpretation of Parameters

Introducing the parameters h_{mc} , h_{ma} , and R_s into the hydrological model enabled reproduction of stepwise infiltration and saturation changes observed in experiments. Table 1 summarizes parameter values for each experiment along with soil characteristics. The results suggest that h_{mc} remains approximately 0.65 across all experiments, indicating that suction-driven water movement is relatively consistent regardless of soil type. In contrast, h_{ma} tends to be higher for soils containing fine particles, such as silty soils, reflecting reduced vertical infiltration due to air entrapment. Additionally, an inverse correlation was observed between R_s and rainfall intensity, implying that higher rainfall rates reduce the relative effect of air exchange on infiltration.

Table 1: Summary of calibrated parameters and soil characteristics across experimental cases

Experimental Case	Soil Type	Parameters			Void Ratio e	Permeability Coefficient (m/s)	Rainfall Intensity mm/h
		h_{ma}	h_{mc}	R_s			
PWRI	Sand soil	0.70	0.65	1.5	0.764	3.65×10^{-5}	20~30
RTRI	Sandy soil mixed with fine sand	0.84	0.65	0.5	0.580	2.98×10^{-5}	30
Kushiro	Sand soil	0.90	0.70	10.0	1.140	7.24×10^{-5}	10
Kushiro	Silty soil	0.80	0.65	-	0.800	1.27×10^{-5}	-

3. Conclusion

This study applied a boundary-fitted hydrological model incorporating saturated and unsaturated flow characteristics to large-scale rainfall experiments on embankments. Using water balance analysis, the model successfully reproduced stepwise infiltration and saturation changes observed in experiments, demonstrating its capability to capture delayed infiltration and

gradual saturation under heavy rainfall. The introduced parameters— h_{mc} , h_{ma} , and R_s —effectively represent unsaturated behavior caused by suction and air entrapment, providing a physically interpretable framework for modeling internal water movement.

However, practical application requires further validation under field conditions, where drainage into foundation layers and soil heterogeneity may significantly affect water dynamics. Future research should establish quantitative relationships between these parameters, soil properties, and rainfall characteristics, enabling broader applicability to diverse embankment types. Additionally, integrating water balance-based boundary conditions with conventional seepage analysis could enhance prediction accuracy for internal moisture distribution, supporting improved design and maintenance strategies for flood-resilient embankments.

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