

APPLICATION OF THREE-DIMENSIONAL NUMERICAL EXPERIMENT RESULTS TO FIELD OBSERVATIONAL DATA OF DEBRIS FLOWS

Hiroki KATO¹, Shoji Fukuoka²

¹ Civil, Human and Environmental Engineering, Chuo University, Japan
email: a17.jnt5@g.chu-u.ac.jp

² Research and Development Initiative, Chuo University, Japan
email: sfukuoka001t@g.chuo-u.ac.jp

ABSTRACT

Clarification of the flow dynamics and development mechanisms of debris flows involving valley bed and bank erosion is an important issue. Observing the movement of water and sediment particles within debris flows is not easy, so in addition to model experiments and field observations, computational mechanics approaches are expected to elucidate the phenomenon. Fukuoka et al. (2014) developed the Arbitrary Particle Multiphase (APM) method, the type of resolved CFD-DEM. Fukuda et al. (2019) used the APM method to explain the vertical distribution of particle velocity and sediment transport concentration, as well as the transport rates of water and sediment, measured in the debris flow channel experiment conducted by Itoh et al. (1999), thereby demonstrating the effectiveness of the numerical model.

The authors conducted three-dimensional numerical experiment on gravel flows using the APM method to investigate the flow dynamics and development mechanisms involving valley bed and bank erosion. Particles smaller than 0.1 m in diameter are not included in this study. The numerical experiment simulates the flow dynamics of a mixture of gravel particles and water. This flow may be referred to as a gravel flow rather than a debris flow. In debris flow phenomena, large particles accumulated in the debris flow head and flowing water give significant impact on the destruction of structures. Therefore, even without explicitly considering the effects of small particles, gravel flow is considered to represent debris flows from an engineering perspective. However, it is necessary to compare numerical experiment results with the actual measured field data to clarify this point. In this study, numerical experiment results were compared with field observation data obtained in other studies.

The results are as follows. First, the velocity distribution of the gravel flow was compared with the field observation results. Large-diameter particles moving within the head section of a gravel flow and within the field debris flow significantly affect the flow. Therefore, in the gravel flow head where large-sized particles accumulate, a linear velocity distribution similar to the field observations by Nagl et al. (2020) was obtained, rather than the convex distribution seen in model experiments (e.g., Lanzoni et al., 2017). Next, the collision forces acting on the valley bed were examined. In field observations, McCoy et al. (2012, 2013) reported time-series data of normal downward forces acting on the valley bed. In this numerical experiment, because a force plate was not installed, collision forces acting on particles on the field valley bed were used as a substitute for force plate measurements. The probability density distribution of the dimensionless collision force for gravel flows closely resembled the field observations of debris flows by McCoy et al. (2013). This indicates that collisions with valley bed particles occur with a probability similar to that of field debris flows.

Based on the comparative analysis conducted in this study between field debris flows and gravel flows—a collective flow of large-sized particles and water—it is considered that the flow mechanisms of field debris flows and gravel flow are mechanically similar.

Keywords: debris flow; Comparison of numerical simulation results with field observations; APM method

1. Introduction

Clarification of the flow dynamics and development mechanisms of debris flows involving valley bed and bank erosion is an important issue. Many model experiments and field observations have been conducted to clarify the constitutive relations and erosion mechanisms of debris flows. In laboratory experiments of debris flows, a convex vertical velocity distribution has been obtained (e.g. Lanzoni et al., 2017). In recent years, the

vertical velocity distribution in debris flows that have actually occurred in the field has been measured. In Nagl et al. (2020), sensor-equipped structures were installed in a field valley, and the flow velocity distribution during the debris flow event was measured. The obtained vertical velocity distribution was linear in the debris flow front and head section where large particles were concentrated. In the trailing section dominated by fine particles forming a mudflow, it was a convex shape. Therefore, it has been clarified that large particles play a significant role in the flow mechanism of field debris flows.

Regarding erosion mechanisms, the particle collision forces acting on valley beds during debris flows were observed and analyzed using force plates installed on the field valley beds (McCoy et al., 2012; McCoy et al. 2013). Their field observations indicate that the maximum collision force can be approximately ten times greater than the time-averaged collision force. In the model experiments conducted by Choi et al. (2023), erosion did not occur unless the force due to particle collisions was at least 4.5% greater than the force due to bed friction. Furthermore, the erosion rate increased with an increasing ratio of particle collision forces to bottom friction forces. Also, particle collision has been shown to be important in the mechanism of bedrock erosion caused by debris flows (e.g. Stock et al., 2006). Based on these findings, particle collision forces are considered to be important in the erosion mechanisms during debris flows.

Berger et al. (2011) used an erosion sensor to measure the total normal stress and pore water pressure acting on the valley bed during a debris flow event, as well as the timing of valley bed erosion. In large debris flows, it was found that valley bed sediment is not eroded at the moment of the debris flow front arrival, but approximately 10 s after arrival, with the eroded sediment being entrained into the head section. Field observations by McCoy et al. (2012, 2013) also showed a similar trend to that reported by Berger et al. (2011).

A lot of data has been obtained through model experiments and field observations. However, because it is difficult to observe the movement of water and sediment particles within debris flows, computational mechanics approaches are expected to clarify the phenomenon, in addition to model experiments and field observations. Fukuoka et al. (2014) developed the Arbitrary Particle Multiphase (APM) method, a type of resolved CFD-DEM. Fukuda et al. (2019) used the APM method to explain the vertical distribution of particle velocity and sediment transport concentration, as well as the transport rates of water and sediment, measured in the debris flow channel experiment conducted by Itoh et al. (1999), thereby demonstrating the effectiveness of the numerical model.

The authors conducted three-dimensional numerical experiments on gravel flows using the APM method to investigate the flow dynamics and development mechanisms involving valley bed and bank erosion. The details of the numerical experimental conditions are described in Chapter 2; however, particles smaller than 0.1 m in diameter are not included in this study. The numerical experiments in this study simulate the flow dynamics of a mixture of gravel particles and water. This flow may be referred to as a gravel flow rather than a debris flow. In debris flow phenomena, large particles accumulated in the debris flow head and flowing water cause the destruction of structures. Therefore, even without explicitly considering the effects of small particles, gravel flow is considered to represent debris flows from an engineering perspective. However, it is necessary to compare numerical results with measured field data to clarify this point. Therefore, in this study, numerical experiment results obtained under the following conditions were compared with field observation data obtained in other studies.

2. Overview of the APM method and numerical experiment conditions

2.1. Overview of the APM method

The APM method analyzes fluid motion using Large Eddy Simulation (LES) and particle motion using the Distinct Element Method (DEM). In flow field calculations, the solid parts are treated as fluids with different densities (solid phase), and within the gravel particles, the flow velocity and pressure are solved as a multiphase flow. Furthermore, the action from particles on the flow field is accounted for through an averaging operation using the volume fraction α of particles in each mesh. The action from the fluid on the particle is evaluated as the fluid force exerted on the particle by the surrounding flow field. At this point, detailed velocity and pressure distributions calculated using a mesh with sufficient resolution for the particles are evaluated by performing volume integration over the solid domain, thereby assessing fluid forces without employing drag coefficients or lift coefficients. It has been demonstrated that such methods yield valid analytical results when grid resolution were appropriately set (e.g. Takiguchi et al., 1998). The governing equations are shown below.

The governing equations for fluid motion are as follows.

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = g_i - \frac{1}{\rho} \frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \{2(\nu + \nu_{SGS})S_{ij}\} \quad (2)$$

$$S_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (3)$$

$$\nu = \mu / \rho \quad (4)$$

$$\nu_{SGS} = (C_s \Delta)^2 \sqrt{2S_{ij}S_{ij}} \quad (5)$$

where u_i is the mass-averaged velocity in the computational grid, P is the sum of isotropic components of pressure and SGS stress, ρ is the volume-averaged density in the computational grid, g_i is the gravitational acceleration, μ is the volume-averaged viscosity in the computational grid, C_s is the Smagorinsky constant, and Δ is the computational grid width. The subscripts i and j denote the components in each direction, respectively.

The governing equations for particle motion are as follows.

$$M \dot{u}_i^g = M g_i + F_i^f + F_i^c \quad (6)$$

$$\dot{\omega}_{l'} = I_{l'j'}^{-1} \{ R_{j'i} (N_i^f + N_i^c) - \varepsilon_{j'k'l'} \omega_{k'} I_{l'm'} \omega_{m'} \} \quad (7)$$

$$F_i^f = \int_{\Omega_s} \left\{ -\frac{\partial P}{\partial x_i} + \rho \frac{\partial}{\partial x_j} \{2(\nu + \nu_{SGS})S_{ij}\} \right\} d\Omega \quad (8)$$

$$N_i^f = \int_{\Omega_s} \varepsilon_{ijk} r_j^f \left\{ -\frac{\partial P}{\partial x_k} + \rho \frac{\partial}{\partial x_l} \{2(\nu + \nu_t)S_{kl}\} \right\} d\Omega \quad (9)$$

where M is the mass, u_i^g is the particle velocity, F is the force on the particle, N is the torque on the particle, ω is the angular velocity, I is the inertia tensor, R is the transformation matrix from the global coordinate system to the local coordinate system, ε is Editon's epsilon, and r_i^f is the distance from the center of gravity of the action. point. The superscript f represents the fluid component and c the solid component. The superscripts $i - k$ denote the global coordinate system and $i' - m'$ denote the local coordinates. Furthermore, the contact force F_i^c is calculated using the Voigt model. For details on the APM method, please refer to the literature (Fukuoka et al., 2014).

2.2. Numerical experiment condition

When conducting experiments on a smaller scale than the field, there is a risk of overestimating viscous shear forces and underestimating the effects of pore water pressure. It has been pointed out that even if the morphology of the experimental results resembles that of the field, the resulting mechanical processes may not necessarily be analogous (Iverson, 2015). Therefore, it is desirable to conduct experiments on as large a scale as possible. Figure 1 shows an overview of this numerical experimental channel. The numerical experimental channel is a large straight channel with a length of 60 m and a width of 8 m. The channel gradient was set to 20°, which is the section where debris flows occur. Additionally, the initial channel was created by randomly depositing particles within the channel such that the slope angle of the valley banks was approximately 30° relative to the valley bed. The initial water surface width is approximately 4.0 meters. Figure 2 shows the size distribution and shape of the

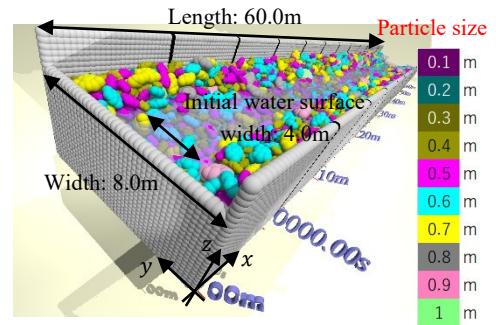


Fig. 1. Overview of this numerical experimental channel

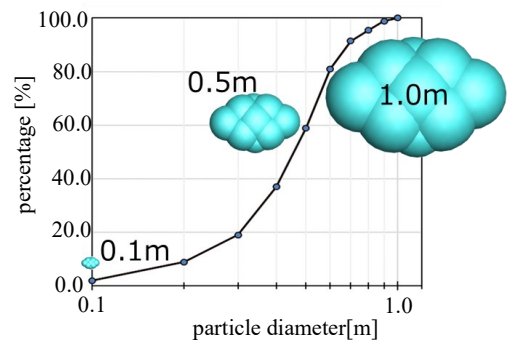


Fig. 2. The particle size distribution and particle shape of the gravel particle

Table 1. Parameters used in the numerical experiment

$\Delta x, \Delta y, \Delta z$: Grid size for fluid calculation	0.025	m
Δt : Time step for fluid calculation	1.0×10^{-4}	s
$\Delta t'$: Time step for particles calculation	2.0×10^{-6}	s
ρ_w : Density of water	1,000	kg/m ³
ρ_s : Density of particles	2,650	kg/m ³
μ_w : Coefficient of viscosity	8.9×10^{-4}	P _a ·s
C_s : Smagorinsky parameter	0.173	
E : Young's modulus	5.0×10^{10}	P _a
ν : Poisson's ratio	0.33	
ϕ_p : Interparticle friction angle	26.6	°
b : Coefficient of restitution	0.7	
Number of grids for fluid calculation	153,600,000	
Initial number of particles	66,619	

gravel particles used in this study. As material for the valley bed and banks, particles with a continuous distribution, with a maximum grain size of 1.0 m and a minimum grain size of 0.1 m were used. Based on the research by Fukuda et al. (2019), it is known that the motion of non-spherical particles differs from that of spherical particles because the particle immediately downstream of the target particle rotates, lifting the upstream particles that it contacts. Furthermore, the motion of particles in a water flow cannot be reproduced by simply varying parameters such as the coefficient of restitution between spherical particles or the interparticle friction angle. Atsumi et al. (2020) demonstrated that for non-spherical particles, differences in particle orientation and arrangement enhance the interlocking effect between particles, making it harder for them to start moving. Consequently, it has been clarified that non-spherical particles exhibit a larger critical shear stress compared to spherical particles. For this reason, this study employs non-spherical particles rather than spherical ones. In this study, non-spherical particles are represented by connecting nine small spheres. Furthermore, all particles are treated as non-spherical particles with identical shape and density, and the size of a non-spherical particle is defined by the diameter of a sphere with the same volume. The parameters used in this numerical analysis are listed in Table 1. These parameters were determined with reference to the study by Fukuda et al. (2019).

In this experiment, a constant discharge of 10.0 [m³/s] was applied at the upstream end to generate a gravel flow. No sediment was supplied to the water at the upstream end. At the downstream end, the pressure was set to a gauge pressure 0 and sediment was allowed to flow out freely. A logarithmic distribution law was applied to the velocity above the valley bed as an initial condition to promote the generation of gravel flows.

3. Comparisons between three-dimensional numerical experiment results and field observation results

3.1. Vertical velocity distribution

Figure 3 shows the vertical distribution of the spatiotemporally averaged flow velocity $\bar{u}$ near the front of the gravel flow. The average velocity $\bar{u}$ is the spatiotemporally averaged velocity in the x-direction within the range $x_{front}(t) - 5\text{m} \leq x \leq x_{front}(t)$ and $-2\text{m} \leq y \leq 2\text{m}$ within the time interval of 5–20 s (Eq. (10)), where x_{front} is the longitudinal position of the gravel flow front. The vertical axis η and the horizontal axis $\bar{u}$ are non-dimensionalized using Eq. (11) and Eq. (12), respectively. Figure 3 also shows the velocity distribution given by Eq. (13) with parameters that align with the experimental values by Lanzoni et al. (2017) and the velocity distribution fitted to the gravel flow. The coefficients for the velocity profile of the gravel flow are $k_0=0.62$ and $\beta=1.12$. The velocity distribution near the front of the gravel flow is linear and has the same characteristics as those of the field-observed velocity distributions of the debris flows.

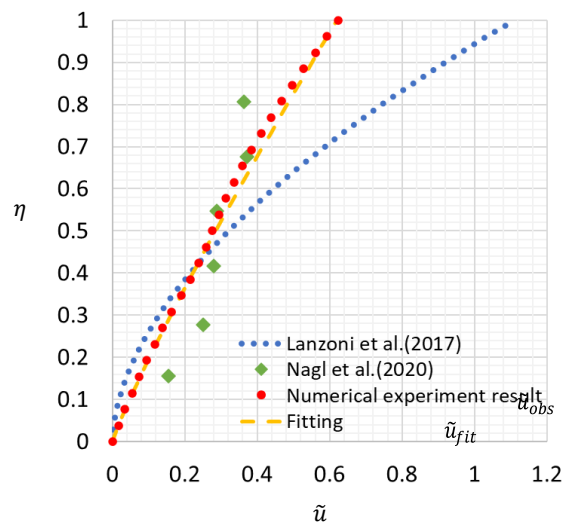


Fig. 3. Vertical velocity distribution of the gravel flows and field debris flows

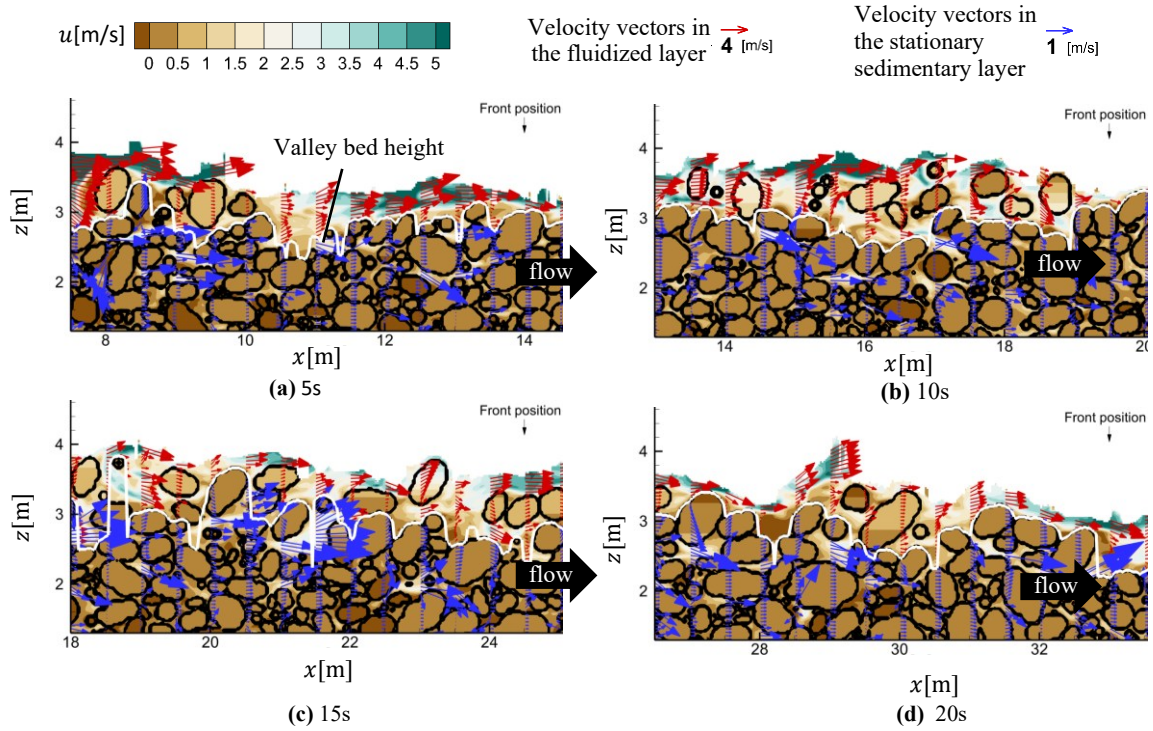


Fig. 4. The contours and vectors of the instantaneous and local velocity u at the head of the gravel flow at the center cross section of the channel

$$\bar{u} = \frac{1}{A} \int_{y=-2}^{y=2} \int_{x=x_{front}-5}^{x=x_{front}} u dx dy \quad (10)$$

$$\eta = \frac{z-z_b}{h} \quad (11)$$

$$\tilde{u}_{obs} = \frac{\bar{u} - \bar{u}_b}{U_0} \quad (12)$$

$$\tilde{u}_{fit} = k_0 \eta^\beta \quad (13)$$

Figure 4 shows the contours and vectors of the instantaneous local velocity u of the gravel flow head at the central cross section of the channel at 5, 10, 15, and 20 s. The closed curves in Fig. 4 indicate particles, and the white lines indicate the height of the valley bed at each time point. The red and blue vectors are velocity vectors in the fluidized and stationary sedimentary layers, respectively; each having a different magnitude range. The velocity inside particles is the velocity of the particle considering both translation and rotation, solved by the DEM. In particular, at $x=18.75$ m and 10 s, the gravel particles are located near the water surface, and the flow velocity is lower near the water surface and higher near the valley bed (Fig. 4b). On the other hand, near $x=15.50$ m, the moving particles are located near the valley bed, and a velocity distribution develops in which the velocity is higher near the water surface and lower near the valley bed. Thus, the velocity distribution inside the gravel flow head is strongly influenced by the gravel particles, indicating that fluid-particle interaction is important when large gravel particles are in motion. In addition, the coarse grain size distribution used in this study and the large pores in the stationary sedimentary layer cause flow velocities in excess of 1.0 m/s even within the stationary sedimentary layer.

3.2. Pressure distribution

Figure 5 shows the temporal variation of the mean pore water pressure and mean total normal stress at $z=2.0$ m and at longitudinal positions of 22–24 m. The gravel flow reached the site at about 12 s. Large-scale model experiment results (Iverson et al., 2010; Reid et al., 2011) show that pore water pressure becomes as large as the total normal stress, and effective stress becomes close to zero when the debris flow arrives. In the numerical gravel flow experiment, both mean pore water pressure and mean total normal stress increased after

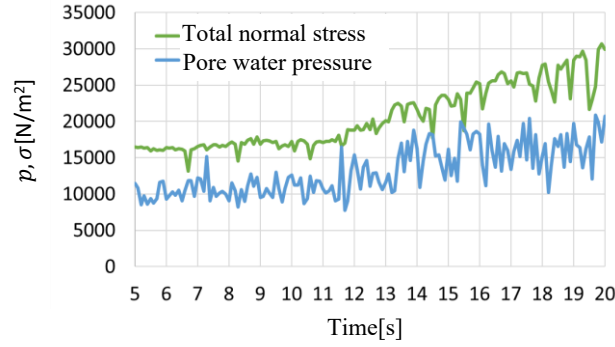


Fig. 5. Temporal variation of the mean pore water pressure and mean total normal stress at $z=2.0$ m at longitudinal positions 22–24 m.

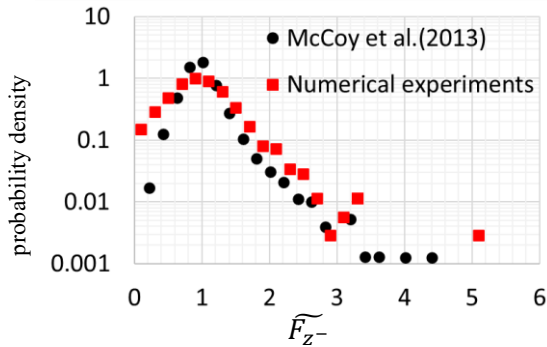


Fig. 6. Probability density distribution of the dimensionless force.

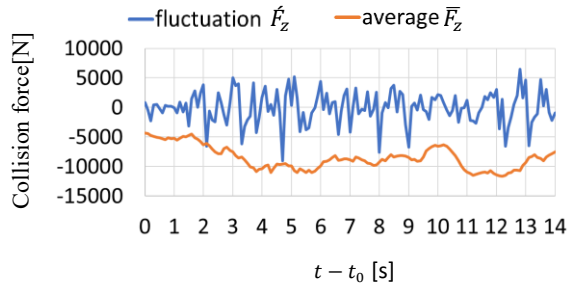


Fig. 7. Time series distribution of contact forces acting on valley bed particles of the interest

approximately 12 s, but the pore water pressure was not as large as the total normal stress even after the arrival of the gravel flow. In this study, because the gravel is coarse and the pore spaces were large, the pore water pressure was therefore not expected to become greater than the total normal stress.

3.3. Probability density distribution of collision forces acting on the valley bed

McCoy et al. (2012, 2013) observed downward forces acting on the valley bed using a force plate installed on the field bed and obtained time-series data. In the numerical experiment, we analyzed the z-directional downward contact force acting on particles that were located near the 15 m point and were not eroded by the gravel flow, as a substitute for the force plate. Figure 6 shows the probability density distribution of the dimensionless force $\tilde{F}_z^c (= F_z^c / \bar{F}_z^c)$ acting on particles. The probability density distribution of the gravel flow is similar to that observed for the field debris flow. Figure 7 shows the time-series distribution of the contact forces acting on the valley bed particles of interest. The particle with initial center-of-gravity coordinates of (15.29, -0.19, 2.34) and a diameter of 0.8 m was selected as the target particle. The average contact force $\bar{F}_z^c(t)$ was obtained by moving averaging the instantaneous contact force $F_z^c(t)$ over 1 s. The fluctuation $\tilde{F}_z^c(t)$ is the difference between the instantaneous contact force and the average value ($F_z^c(t) - \bar{F}_z^c(t)$). The magnitude of $\tilde{F}_z^c(t)$ obtained from the numerical experiments of the gravel flow is about 20 times larger than that of the field observations. The grain size distribution used in this numerical experiment is coarser than that in the field. Therefore, the numerical results show larger contact forces than the field observations. McCoy et al. (2013) observed that the magnitude of the fluctuation is about 10 times the time-averaged value, but the amount of fluctuation of the contact force in the gravel flow is not large. This may be due to the low velocity of the gravel flow and the fact that the ratio of the maximum grain size $d_{\max}$ to minimum grain size $d_{\min}$ is 10 in gravel flows, whereas in field debris flows, $d_{\max}/d_{\min}$ exceeds 10^4 .

3.4. Timing of valley bed erosion

Figure 8 shows the relationship between the timing of valley bed erosion and the front position, as observed in field measurements by Berger et al. (2011) and in numerical gravel flow results. The horizontal and vertical axes were non-dimensionalized by dividing by the maximum time and longitudinal distance of the tracked

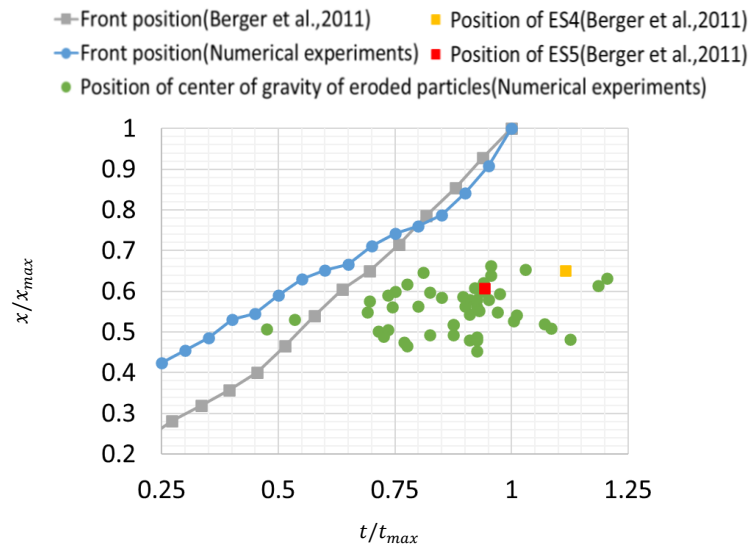


Fig. 8. Relationship between the timing of valley bed and bank particles erosion and the temporal variation of the front position.

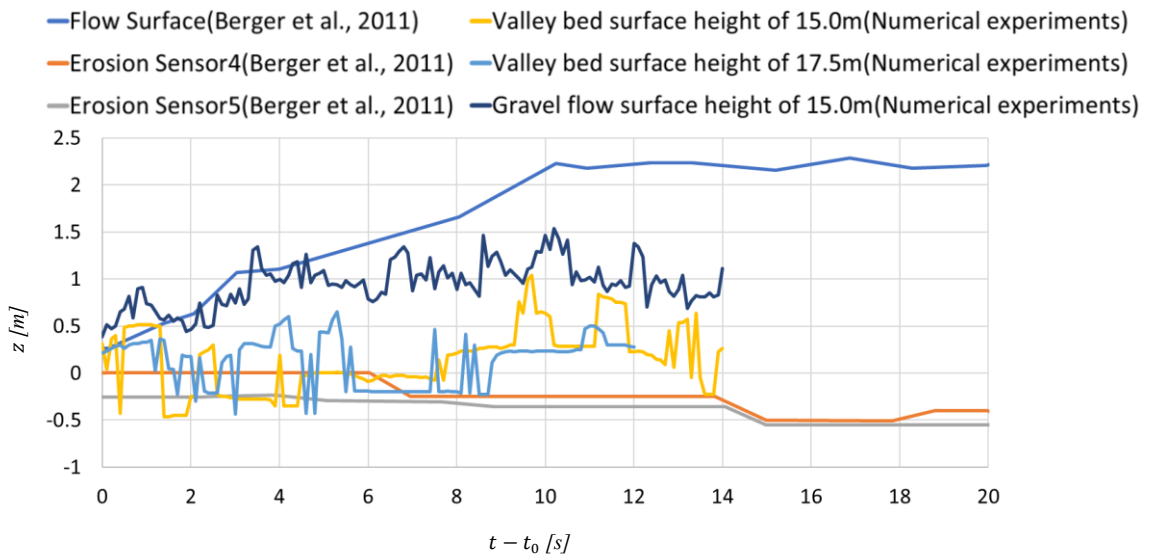


Fig. 9. Time variation of valley bed height.

debris flow front position. The t_{max} of the numerical experimental is 20 s. The timing of the erosion of the valley bed and banks in the gravel flow is defined as the moment when each of these target particles begins to move. In both the numerical gravel flow and field debris flows, erosion occurs approximately 5 to 10 s after the front arrives. Figure 9 shows the temporal variation of water level and valley bed height observed in the field debris flow and the temporal variation of water level and valley bed height in the longitudinally positioned gravel flow. t_0 is the time at which the front of both the gravel flow and the debris flow reached the observation point. Field observation results (Berger et al., 2011) indicate that the valley bed was being eroded in stages. On the other hand, the numerical experiment results show repeated cycles of erosion and deposition. In the observations by Berger et al. (2011), the timing and depth of erosion were measured by observing changes in circuit resistance caused by erosion detected by the erosion sensor. Even if deposition occurs after erosion, it does not change the circuit resistance; therefore, this observation method cannot detect deposition and the remobilization of deposited sediment (Berger et al., 2010). The observation results of Berger et al. (2011) yield such results because this method cannot observe deposition and remobilization. The actual debris flows are thought to descend while repeatedly undergoing erosion and deposition, as seen in the numerical experiment results.

4. Summary

In this study, three-dimensional numerical experiments using the APM method were compared with field observation results reported by other researchers. As a result, it was found that the characteristics of the gravel flow and the field debris flows were largely the same. In particular, similar trends were observed for the probability density distribution of collision forces and the timing of valley bed erosion in both gravel flows and field debris flows. This indicates that large-sized particles dominate both collision forces—a driving force for erosion—and skeletal stress, which plays a significant role as resistance. Furthermore, previous studies by the authors have clarified that the depth of the debris flow head is comparable to the grain size of the large particles accumulated at its front. Consequently, the depth and flow structure of field debris flows are determined by the size of these large particles. Similarly, the depth of the gravel flow head is comparable to the size of the large particles accumulated at the front, and this indicates that the flow structure is strongly influenced by these large particles. These findings show that the gravel flow, which is a collective flow composed of large particles and water, is capable of representing the key mechanisms of field debris flows.

References

- Atsumi, T. & Fukuoka, S. (2020). Effects of irregular particle shapes on the sediment movement and transport rate in gravel-bed channels. *River Flow 2020*, 294-301. <https://doi.org/10.1201/b22619>
- Berger, C., McArdell, B.W., & Schlunegger, F. (2011). Direct measurement of channel erosion by debris flows, Illgraben, Switzerland. *Journal of Geophysical Research*, 116, F01002. <https://doi.org/10.1029/2010JF001722>
- Berger, C., McArdell, B.W., Fritshi, B. & Schlunegger, F. (2010). A novel method for measuring the timing of bed erosion during debris flows and floods. *Water Resources Research*, 46, W02502. doi:10.1029/2009WR007993
- Choi, C.E. & Song, P. (2023). New unsaturated erosion model for landslide: Effects of flow particle size and debunking the importance of frictional stress. *Engineering Geology*, 315, 20, 107024. <https://doi.org/10.1016/j.enggeo.2023.107024>
- Fukuda, T. & Fukuoka, S. (2019). Interface-resolved large eddy simulations of hyper-concentrated flows using spheres and gravel particles. *Advances in Water Resources*, 129, 297–310. <https://doi.org/10.1016/j.advwatres.2017.10.037>
- Fukuoka, S., Fukuda, T., & Uchida, T. (2014). Effects of sizes and shapes of gravel particles on sediment transports and bed variations in a numerical movable-bed channel. *Advances in Water Resources*, 72, 84–96. <https://doi.org/10.1016/j.advwatres.2014.05.013>
- Itoh, T., Egashira, S., Miyamoto, K., & Tkeuchi, H. (1999). Transition of debris flows over rigid beds to over erodible beds. *Annual journal of hydraulic engineering*, 43, 635–640. <https://doi.org/10.2208/prohe.43.635>
- Iverson, R.M. (2015). Scaling and design of landslide and debris-flow experiments. *Geomorphology*, 244, 9-20. <https://doi.org/10.1016/j.geomorph.2015.02.033>
- Iverson, R.M., Logan, M., LaHusen, R.G., & Berti, M. (2010). The perfect debris flow? Aggregated results from 28 large-scale experiments. *Journal of Geophysical Research*, 115, F03005. <https://doi.org/10.1029/2009JF001514>
- Lanzoni, S., Gregoretti, C., & Stancanelli, L.M. (2017). Coarse-grained debris flow dynamics on erodible beds. *Journal of Geophysical Research: Earth Surface*, 122, 592–614. <https://doi.org/10.1002/2016JF004046>
- McCoy, S.W., Kean, J.W., Coe, J.A., Tucker, G.E., Staley, D.M., & Wasklewicz, T.A. (2012). Sediment entrainment by debris flows: In situ measurements from the headwaters of a steep catchment. *Journal of Geophysical Research*, 117, F03016. <https://doi.org/10.1029/2011JF002278>
- McCoy, S.W., Tucker, G.E., Kean, J.W., & Coe, J.A. (2013). Field measurement of basal forces generated by erosive debris flows. *Journal of Geophysical Research: Earth Surface*, 118, 589–602. <https://doi.org/10.1002/jgrf.20041>
- Nagl, G., Hübl, J., & Kaitna, R. (2020). Velocity profiles and basal stresses in natural debris flows. *Earth Surface Processes and Landforms*, 45, 1764–1776. <https://doi.org/10.1002/esp.4844>
- Reid, M.E., Iverson, R.M., Logan, M., LaHusen, R.G., Godt, J.W., and Griswold, J.P. (2011). Entrainment of Bed Sediment by Debris Flows: Results from Large-Scale Experiments, *Italian Journal of Engineering Geology and Environment*, 367 – 374 . <https://doi.org/10.4408/IJEGE.2011-03.B-042>
- Stock, J.D. & Dietrich, W.E. (2006). Erosion of steepland valleys by debris flows. *Geological Society of America Bulletin*, 118, 1125-1148. <https://doi.org/10.1130/B25902.1>
- Takiguchi, A., Kajishima, T. and Miyake, Y. (1998). Numerical scheme to resolve the interaction between solid-particles and fluid-turbulence. *Transactions of the Japan Society of Mechanical Engineers, B*, 64, 48–54. <https://doi.org/10.1299/jsmeb.42.411>