

# Numerical calculation for bed variation in compound-meandering channel using depth integrated model without assumption of shallow water flow

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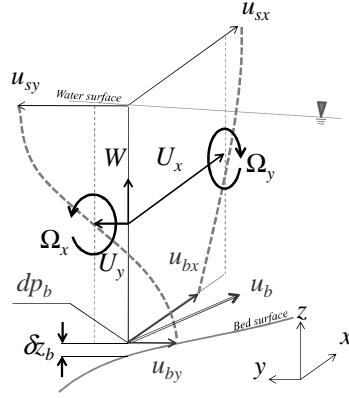
*In a compound-meandering channel, patterns of flow structures and bed variations change with complex momentum exchanged between high velocity flow in a main channel and low velocity flows in flood plains due to the increase of the water depth. A method to calculate these hydraulic phenomena in the compound-meandering channel is required to clarify mechanisms of bed scouring and bank erosions during a flood event. We have developed a new depth integrated model without the shallow water assumption such as the hydrostatic pressure distribution, the general Bottom Velocity Computation (BVC) method, in which a set of depth integrated equations including depth integrated momentum and vorticity equations are prepared for evaluating bottom velocity and vertical velocity distributions. The objective of this paper is to develop a bed variation calculation method for both single-meandering and compound-meandering channels by using the general BVC method coupled with sediment transport model. This paper presents that the general BVC method can reproduce the pattern change of bed variation in a compound meandering channel flow with increasing relative depth.*

## **Key words**

Compound meandering channel, bed variation, secondary flow, depth integrated model, shallow water assumption, general bottom velocity computation method.

## **I INTRODUCTION**

In a compound-meandering channel, the changes in patterns of flow structures and bed variations are induced by increasing water depth on flood plains. It has been considered as the results of the complex momentum exchange with 3D vortex motions between high velocity flow in a main channel and low velocity flows in flood plains [2]. A calculation method to explain these phenomena in the compound-meandering channel is required to clarify mechanisms of bed scouring and bank erosions in rivers during flood events. Until now, researchers and engineers have improved depth integrated models, especially for a helical flow in a curved or meandering channel. Most of previous advanced depth integrated models have been employed the assumption of the hydrostatic pressure distribution. However, Watanabe & Fukuoka [6] showed that the non-hydrostatic pressure distribution along the bank of the main channel played an important role to calculate bed variation in a compound-meandering channel. Although we have become to simulate complex turbulent flow and local scouring around a pier by a recent advanced 3D turbulence model [3], applications of the 3D turbulence model



Vertical velocity distribution:  
 $u_i = U_i + \Delta u_i (12\eta^2 - 12\eta + 1) - \delta \Omega_i (4\eta^3 - 3\eta^2)$ .

Depth-integrated continuity eq.: $h$	3D model	BVC method with SWA	General BVC method
DA horizontal momentum eq.: $U_i$			
DA turbulence energy transport eq.: $k$			
Bottom velocity eq.: $u_{bi}$			
DA horizontal vorticity eq.: $\Omega_i$			
Horizontal momentum eq. on water surface: $u_{si}$			
Poisson eq. for time variation in DA vertical velocity: $W$			
DA vertical momentum eq. for bottom pressure intensity: $dp_b$			

DA: Depth Averaged, BVC: Bottom Velocity Computation, SWA: Shallow Water Assumption  $\Delta u_i = U_i - U_i$ ,  $\delta \Omega_i = \Omega_i - \Omega_i$

**Fig.1.** Governing equations and unknown quantities of general BVC method

are still limited to small scale phenomena such as local scouring around structures in experimental channels because of large computational time, many memories, and a lot of computational task. In fact, 3D numerical calculation to the bed variation analysis in a compound meandering channel is still challenging [6][7].

To dissolve the limitation of the previous depth integrated models, we have developed a new depth integrated model (general Bottom Velocity Computation method) without the shallow water assumption such as the hydrostatic pressure distribution, the Bottom Velocity Computation (BVC) method, in which equations of 3D motions are computed with shallow water equations to evaluate bottom velocity and vertical velocity distributions [5]. The objective of this paper is to develop a bed variation calculation method both for single-meandering and compound-meandering channels by using the general BVC method coupled with sediment transport model.

## II CALCULATION METHOD

For bed variation analysis, we have to evaluate bed tractive forces acting on sediment particles by a hydrodynamic model. The general BVC method makes it possible to calculate the bottom velocity acting on bed surface by the bottom velocity equation. The equation is derived by depth-integrating horizontal vorticity:

$$\delta u_i = u_{si} - u_{bi} = \varepsilon_{ij3} \Omega_j h + \frac{\partial W h}{\partial x_i} - w_s \frac{\partial z_s}{\partial x_i} + w_b \frac{\partial z_b}{\partial x_i} \quad (1)$$

Where,  $u_{bi}$ : bottom velocity,  $u_{si}$ : water surface velocity,  $\Omega_j$ : depth averaged (DA) horizontal vorticity,  $h$ : water depth,  $W$ : DA vertical velocity,  $z_s$ : water level,  $z_b$ : bed level,  $w_s$ ,  $w_b$ : vertical velocity on water surface and bottom. The bottom shear stress acting on the bed  $\tau_{bi}$  is evaluated with equivalent roughness  $k_s$ , assuming the equilibrium velocity condition in the bottom layer  $\delta z_b$ .

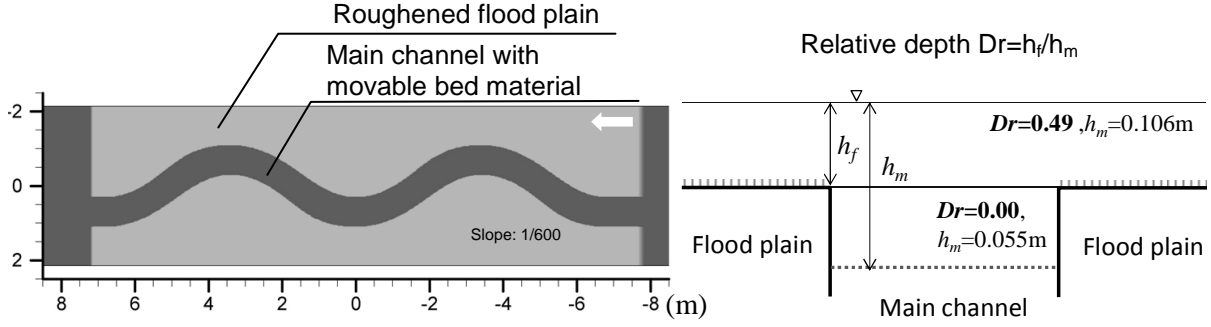
$$\tau_{bi} = (c_b)^2 u_i u_b, \quad \frac{1}{c_b} = Ar + \frac{1}{\kappa} \ln \left( \frac{\delta z_b + a k_s}{k_s} \right) \quad (2)$$

Where,  $h/\delta z_b = e^3 - 1$ ,  $a$ : parameter for origin height of the log-law. On the other hand, the bottom pressure intensity is given by Eq.(3), depth-integrating the vertical momentum equation.

$$\frac{dp_b}{\rho} = \frac{\partial h W}{\partial t} + \frac{\partial h W U_j}{\partial x_j} + \tau_{bj} \frac{\partial z_b}{\partial x_j} - \frac{\partial h \tau_{zj}}{\partial x_j} \quad (3)$$

Where,  $dp$ : pressure deviation from hydrostatic pressure distribution ( $p = \rho g(z_s - z) + dp$ ),  $dp_b$ :  $dp$  on bottom,  $\tau_{bj}$ : bed shear stress,  $\tau_{ij}$ : shear stress tensor due to molecular and turbulence motions and vertical velocity distribution. To calculate Eqs. (1) and (3), the governing equations for the unknown quantities are solved in the general BVC method as shown in **Fig.1**. Refer to the literatures [5] for details of the equations and the numerical computation method for the general BVC method.

The time variation in bed level is calculated by the 2D continuity equation for sediment transport of the Exner's form.



**Fig.2.** Compound meandering channel and calculation conditions

$$(1 - \lambda_B) \frac{\partial z_b}{\partial t} + \frac{\partial q_{Bi}}{\partial x_i} = 0 \quad (4)$$

Where,  $\lambda_B$ : sediment porosity,  $q_{Bi}$ : bedload sediment transport rate. The sediment transport rate is calculated by the momentum equation for sediment motion to take into account non-equilibrium motions of sediment particles [4] :

$$\frac{\partial q_{Bi}}{\partial t} + \frac{\partial u_{Bj} q_{Bi}}{\partial x_j} = (Pu_{BPi} - Du_{BDi}) + m_* h_B (\gamma_{ei} - \gamma_i) \quad (5)$$

Where,  $u_{Bi}$ : sediment particle velocity of bedload,  $Pu_{BPi}$ : gain momentum from bed material,  $Du_{BDi}$ : loss momentum by particle deposition,  $m_* = \mu_k s g \cos \theta / (s + 1 + C_M)$ ,  $C_M$ : coefficient of added mass,  $\mu_k$ : dynamic friction coefficient of sediment,  $s$ : specific gravity of sediment in water,  $\theta$ : maximum bed gradient,  $h_B = q_B / u_B$ ,  $q_B = q_{Bi} q_{Bi}$ ,  $u_B = u_{Bi} u_{Bi}$ ,  $\gamma_i$ ,  $\gamma_{ei}$ :  $i$  component of unit vector of sediment movement and its equilibrium value. The first term in right side for momentum exchange between bed load and bed materials is described as:

$$Pu_{BPi} - Du_{BDi} = q_{Be} u_{Bei} / L_e - q_B u_{Bi} / L \quad (6)$$

Where,  $q_{Be}$ : equilibrium sediment transport rate of bed load,  $L$ : average spatial lag,  $L_e$ :  $L$  for equilibrium condition,  $q_{Be}$ ,  $u_B$ ,  $u_{Be}$ ,  $h_B$  are given by reference to Ashida & Michiue formula [1]. The gravity effects on bed tractive force and the critical shear stress of sediment particle on bed slope are taken into account. Where the local bed slope exceeds the angle of repose, the cross-section changes in shape due to sliding. Refer to the literatures [4] for details of bed variation analysis.

### III APPLICATION OF THE GENERAL BVC METHOD TO BED VARIATION IN A COMPOUND MEANDERING CHANNEL

In this section, we apply the general BVC method coupled with a non-equilibrium sediment transport model to bed-variation analysis in a compound meandering channel. **Fig.2** shows a compound meandering channel and calculation conditions. Fukuoka et al. [2] investigated the bed variations in meandering compound channel flows with different relative depths  $Dr$ . The present method is applied to their experimental results. The channel has a meandering main channel with two wavelengths of the sine-generated curve and wide straight flood plains.

**Fig.3** shows comparisons of bed variations between measured and computed results in the flows of  $Dr=0.0$  and  $Dr=0.49$ . For a simple meandering channel ( $Dr=0.0$ ), we can see large scours along the outer bank and large point bars along the inner bank. However, the increment in the relative depth of a compound meandering channel causes inner bank scours instead of the outer bank scour, decreasing the area of point bars. The reason of the above hydraulic phenomena is because the helical flow structure in a compound meandering channel is different from that in a simple meandering channel due to 3D flow interactions between a main-channel and flood plains [2]. The present calculation results give a better agreement with experimental results rather than full-3D calculation results with the periodic boundary condition [6]. This proves that the actual boundary conditions should be given for the bed variation analysis, because the periodic boundary condition would give an

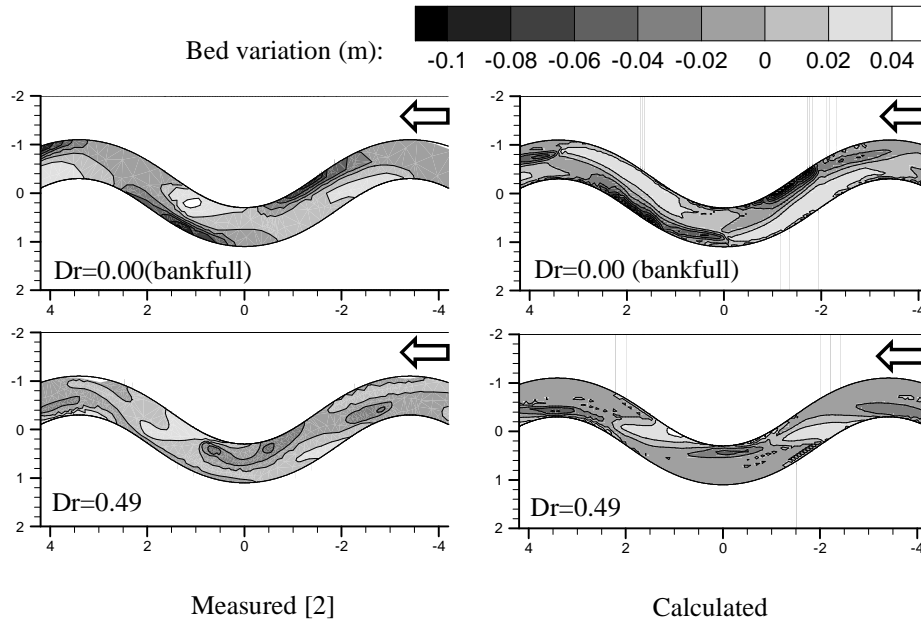


Fig.3 Bed variations in a compound meandering channel for two different types of flow

unrealistic assumption for the analysis. Therefore it would be safe to say that the general BVC method can reproduce bed variations in compound meandering channel flows with increasing relative depth.

#### IV CONCLUSIONS

This paper developed a bed variation analysis method in a compound meandering channel by using the general BVC method based on the depth integrated model. The calculated result showed large scours along the outer bank for the bankfull flow condition ( $Dr=0.00$ ) and inner bank scours for the overbank flow condition ( $Dr=0.49$ ). The results show that general BVC method is able to calculate the pattern change in the secondary flow structure and bottom velocity distribution by increasing the water depth of the overbank flow.

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