A Depth Integrated Model for 3D Turbulence Flows Using Shallow Water Equations and Horizontal Vorticity Equations

T. Uchida[1] and S. Fukuoka[2]

[1]Research and Development Initiative, Chuo University, 1-13-27 Kasuga, Bunkyo-ku, Tokyo, 112-8551, Japan; Phone: +81-3-3817-1617; Email: utida@tamacc.chuo-u.ac.jp

[2]Research and Development Initiative, Chuo University, 1-13-27 Kasuga, Bunkyo-ku, Tokyo, 112-8551, Japan, Phone: +81-3-3817-1625; Email: sfuku@tamacc.chuo-u.ac.jp

ABSTRACT

There is a pressing need to check the safety degree of rivers, especially in channel confluence sections, in which several problems will be occurred during floods such as water level rising and bed scouring. A practical and reliable model for flows and bed variations in channel junctions are highly required. This study proposes a new methodology of computing velocity distributions on bed without the assumption of the hydrostatic pressure distribution in order to develop a bed variation model for complex flow fields such as flows around a channel confluence and hydraulic structures. First, we introduce depth-integrated momentum equations and a turbulence energy transport equation with terms of deviation component velocities from the depth averaged value by using a parabolic velocity distribution. Then, depth-integrated horizontal vorticity equations are derived to evaluate the deviation component velocities. The adequacy of the model is discussed through the comparisons with the experimental results for flows in a channel confluence.

INTRODUCTION

A safety degree of a river in Japan will be significantly reduced by the global warming. There is a pressing need to check the safety degree of rivers, especially in channel confluence sections, in which several problems will be occurred during floods such as water level rising and bed scouring.

The previous studies for channel junctions are divided into two categories, flows on fixed bed and bed variation (including channel deformation). The former category includes many experimental studies for 3D flow structures with separation (e.g. Fujita & Komura, 1990; Biron *et al.*, 1996; Weber *et al.*, 2001). One of prior studies on numerical analysis for flows with a channel confluence was presented by Mcguirk & Rodi (1978). The model consists of depth averaged continuity and momentum equations with k-ε model. A 3D turbulence model is generally required to calculate flow configurations due to channel junctions. Recently, 3D numerical

33rd IAHR Congress: Water Engineering for a Sustainable Environment

ISBN: 978-94-90365-01-1

analysis have been conducted (e.g. Weerakoon & Tamai, 1989; Huang *et al.*, 2002). However, applications of the 3D model are still limited for floods in natural rivers. So a practical and reliable model for flows and bed variations in channel junctions are highly required.

2D numerical model have been used for flows and bed variations in rivers during flood events. Improved 2D model have been developed to evaluate the helical flow due to the centrifugal force in curved and meandering channels. Especially, bed surface velocity is important for bed variation analysis. The conventional evaluation method of bed surface velocity due to stream curvatures (Engelund, 1974), full-developed secondary flow model without considering effects of secondary flow on vertical distribution of stream-wise velocity, has been applied to bed variation analyses in channels (e.g. Nishimoto *et al.*, 1992). Refined secondary flow models can consider non-equilibrium (e.g. Ikeda & Nishimura, 1986; Finnie *et al.*, 1999) and mean flow redistribution effects (e.g. Blanckaert & de Vriend, 2003; Onda *et al.*, 2006). As more versatile method for evaluating vertical velocity distributions, a quasi-3D model, in which equations for vertical velocity distribution are computed, has been developed (e.g. Ishikawa *et al.*, 1986; Fukuoka *et al.*, 1992; Jin & Steffler, 1993; Yeh & Kennedy, 1993). The above mentioned models are categorized in the efficiently-simplified 3D model with the assumption of hydrostatic pressure distribution for each objective of the computation. However, it is known that these models are still-inadequate for local scouring due to complex flows such as flows around a channel confluence and hydraulic structures, because they cannot consider effects of flows due to non-hydrostatic pressure distribution on bed scouring (e.g. the downward flow in front of structures). So, a reliable depth integrated models with considering vertical velocity distributions due to non-hydrostatic pressure distribution is highly required.

This study proposes a new methodology of computing velocity distributions on bed without the assumption of the hydrostatic pressure distribution in order to develop a bed variation model for complex flow fields such as flows around a channel confluence and hydraulic structures. First, we introduce depth-integrated momentum equations and a turbulence energy transport equation with terms of deviation component velocities from the depth averaged value by using a parabolic velocity distribution. Then, depth-integrated horizontal vorticity equations are derived to evaluate the deviation component velocities. The adequacy of the model is discussed through the comparisons with the experimental results (Weber *et al.*, 2001) for flows in a channel confluence.

GOVERNING EQUATIONS AND COMPUTATIONAL SCHEMES

Vertical velocity distribution. Velocity distributions on very thin layer on bed surface, the idea is after the slip-velocity model (Engelund, 1974), are discussed in this paper. The quadric curve (1) of vertical velocity distributions are obtained by using depth averaged velocity U_i and slip-velocity u_{bi} (velocity on the thin layer, z=z_b), if vertical velocity gradients on water surface (z=z_s) are zero:

$$u'_i = u_i - U_i = \frac{\delta u_i}{2}\left(1-3\eta^2\right) = \frac{U_i - u_{bi}}{2}\left(1-3\eta^2\right) \tag{1}$$

where u_i=i direction velocity, η=$(z_s-z)/h$, h=z_s-z_b, water depth.

For the equilibrium condition (uniform flow), the following equations are obtained. With the eddy-viscosity model, shear stress τ_{zi} is written as:

$$\tau_{zi} = \tau_{0i}{}^2\eta = \nu\frac{du'_i}{dz}\eta = \frac{3\nu\delta u_i}{h}\eta \tag{2}$$

where τ_{0i}=i direction bed shear stress defined as:

$$\tau_{0i} = C_0{}^2 U_i\sqrt{U_j{}^2} = c_b{}^2 u_{bi}\sqrt{u_{bj}{}^2} \tag{3}$$

δu_i for the equilibrium condition is given by:

$$\delta u_i = C_0{}^2 U_i\sqrt{U_j{}^2}\,\frac{h}{3\nu} = c_b{}^2 u_{bi}\sqrt{u_{bj}{}^2}\,\frac{h}{3\nu} \tag{4}$$

With the depth averaged eddy-viscosity ν=$\alpha u_* h$, α=κ/6, the relationship between c_b and C_0 is described as:

$$c_b = C_0/\left(1-2C_0/\kappa\right) \tag{5}$$

In this paper, C_0 is described by manning roughness coefficient:

$$C_0{}^2 = gn^2/h^{1/3} \tag{6}$$

Depth averaged velocity vectors and water depth. Governing equations for water depth h and depth averaged velocity vectors U_i (shallow water equations) with occupancy ratio of water λ described as:

$$\frac{\partial\lambda h}{\partial t} + \frac{\partial U_j\cdot\lambda h}{\partial x_j} = 0 \tag{7}$$

$$\frac{\partial\lambda U_i h}{\lambda h\partial t} + \frac{\partial\lambda U_i U_j h}{\lambda h\partial x_j} = -g\frac{\partial z_s}{\partial x_i} - \frac{\tau_{0i}}{h} - \frac{\tau_{swi}}{R_{sw}} + \frac{\partial\lambda h\tau_{ij}}{\lambda h\partial x_j} \tag{8}$$

where τ_{swi}=shear stress on side wall, R_{sw}=hydraulic radius of side wall. τ_{ij} is horizontal shear stress:

$$\tau_{ij} = 2\nu S_{ij} - \delta u_i\delta u_j/5 - 2/3\cdot\delta_{ij}\cdot k \tag{9}$$

where k=depth averaged turbulence energy, S_{ij}=strain ratio tensor of depth averaged velocity vectors U_i. k is computed by Eq.(10), which is known as one-equation model (Rodi, 1984):

$$\frac{\partial k}{\partial t} + u_j\frac{\partial k}{\partial x_j} = \frac{1}{h\lambda}\frac{\partial}{\partial x_i}\left(\frac{\nu h\lambda}{\sigma_k}\frac{\partial k}{\partial x_i}\right) + \frac{D_{sw}}{R_{sw}} + P_k - \varepsilon \tag{10}$$

where ν=ν_m+$C_\mu k^2/\varepsilon$, ε=$C_\varepsilon k^{3/2}/\Delta$, C_μ=0.09, C_ε/Δ=1.7/h (Nadaoka & Yagi, 1984). With using vertical velocity distribution (1), the production term P_k is derived as:

$$\frac{P_k}{\nu} = 2\left(S_{ij}{}^2 + \frac{1}{5}\delta S_{ij}{}^2\right) + C_h\left(\frac{\delta u_i}{h}\right)^2 \tag{11}$$

where δS_{ij}=strain ratio tensor of δu_i(=U_i-u_{bi}, see Eq.(1)). C_h=9$(\alpha C_\varepsilon)^4/C_\mu{}^3$≈2.25. The model is equivalent of zero-equation model for the equilibrium condition of k.

Depth averaged horizontal vorticity vectors. With Stokes' theorem, δu_i is described as:

$$\delta u_i = \frac{2}{3}\left(\varepsilon_{ij3}\Omega_j h + \frac{\partial Wh}{\partial x_i} \right) \approx \frac{2}{3}\varepsilon_{ij3}\Omega_j h \tag{12}$$

where ε_{ij3}=permutation symbol ($\varepsilon_{123}= -\varepsilon_{213}=1$, $\varepsilon_{113}= \varepsilon_{223}=0$), W=depth averaged vertical velocity, Ω_i =depth averaged vorticity vectors. Ω_i is described by:

$$\frac{\partial \lambda \Omega_i h}{\lambda \partial t} + \frac{\partial \lambda U_j \Omega_i h}{\lambda \partial x_j} = ER_i + P_{\omega i} + \frac{\partial \lambda h D_{\omega ij}}{\lambda \partial x_j} \tag{13}$$

where,

$$D_{\omega ij} = \frac{\nu}{\sigma_\omega}\frac{\partial \Omega_i}{\partial x_j} + \frac{\delta u_j \Omega_i}{4}, \ \Omega_z = \frac{\partial U_y}{\partial x} - \frac{\partial U_x}{\partial y}, \quad \delta\omega_z = \frac{\partial \delta u_y}{\partial x} - \frac{\partial \delta u_x}{\partial y}$$

$$ER_i = \Omega_j \frac{\partial U_i h}{\partial x_j} - \Omega_j \left(u_{si}\frac{\partial z_s}{\partial x_j} - u_{bi}\frac{\partial z_b}{\partial x_j} \right) - \frac{\Omega_j h}{4}\frac{\partial \delta u_i}{\partial x_j} + \frac{3}{2}\delta u_i \left(\Omega_z - \frac{\delta\omega_z}{4} \right)$$

$P_{\omega i}$ =production term of vorticity vectors. With wall law of eddy viscosity (Rodi, 1984), $P_{\omega i}$ is approximated by:

$$P_{\omega i} = 2C_{p\omega}\nu_{b0}\frac{\Omega_{ei} - \Omega_i}{h} \tag{14}$$

where,

$$\Omega_{ei} = -\frac{3c_b \varepsilon_{ij3} u_{bj}}{\kappa h}, \ \nu_{b0} = (\alpha h)^2 2\sqrt{\Omega_{bi}{}^2}, \ \Omega_{bi}{}^2 = \max(\Omega_i{}^2, \Omega_{ei}{}^2), \ C_{p\omega}=\kappa/\alpha.$$

Numerical schemes. The explicit conservative CIP scheme for shallow water flows (Uchida & Kawahara, 2006) is adopted for computations of water depth and depth averaged velocity vectors. The important points of the scheme are as follows. In the model, to directly capture the effect of distributed parameters in the Cartesian coordinate system, each control volume ij has three kinds of variables, i.e., the value at the intersection of the grid (Point Value), the averaged value along the side of the grid (Line-averaged Value) and the averaged value over the grid (Area-averaged Value). Those values are computed all together based on CIP-CSL scheme (Nakamura *et al.*, 2001). All the variables in the governing equations are set on the same location. The interpolating operation of the variables is not necessary in the present scheme. Any boundary conditions can be taken into account without difficulty, because all the variables and parameters are well-defined in a computational cell based on the Control Volume Approach. The utilization of multi-valuables on a computational cell enables us not only to capture complex geometry even on the Cartesian coordinate system, but also to compute flow transitions with high resolution and accuracy. Details of numerical method and its performances are discussed in our previous paper (e.g. Uchida & Kawahara, 2006; Uchida *et al.*, 2007). The above scheme is not adopted for computations of the depth averaged turbulence energy and vorticity vectors, only those averaged value of the computational cell are computed by traditional scheme. Because, shallow water equations control flow fields rather than depth averaged turbulence energy equation and vorticity vectors equations, the main role of those equations is to decide horizontal shear stress terms of Eq.(8). And the accuracy of diffusion terms and production terms of Eq.(10) and (13) is not high, compared with those of shallow water equations.

APPLICATION TO FLOWS IN A CHANNEL CONFLUENCE

Computational conditions. In this chapter, the present model is applied to flows with a right angle channel confluence experiment (Weber *et al.*, 2001), as shown in Figure 1. For the constant total discharge Q=0.170 m^3/s and channel width B=0.914m, water level and velocity vectors were measured for various discharge ratio Q^*. In the computation, discharges are given at upstream ends of the main channel and the tributary channel. Water levels are given at downstream end of the main channel. The computational grid is shown in Figure 1, the grid size is 1/15 of channel width B.

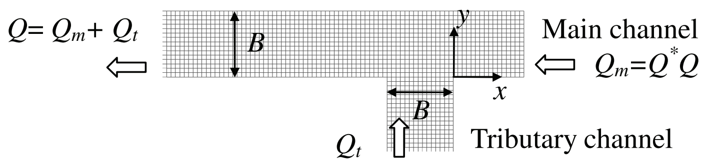
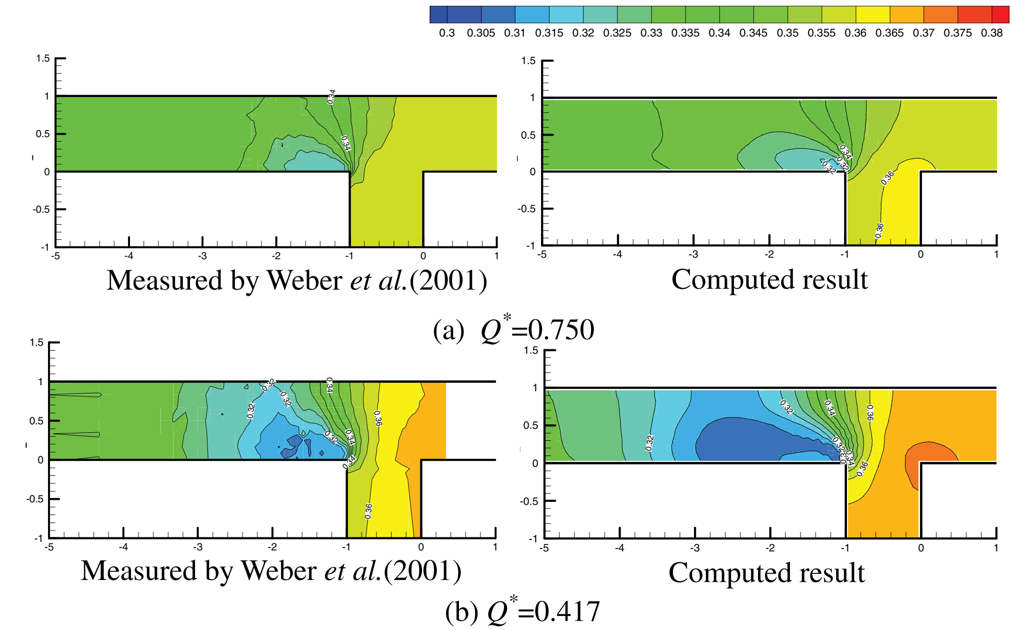


Figure 1 Computational grid around a channel junction

Computational results and discussions. Figure 2 and 3 show comparisons respectively of water level profiles and velocity vectors between measured (Weber *et al.*, 2001) and computed results for Q^*=0.750, 0.417. The following is the characteristics of flows with a channel confluence. A separation zone develops and the water level here decreases just downstream from the tributary channel with increasing the tributary discharge (i.e. decreasing Q^*). Two helical flows are created in water paths from the main channel and the tributary channel. With increasing the tributary discharge, bed surface velocity vectors are bended to the left bank of the main channel due to the helical flow in the tributary channel water. And bed surface velocity is accelerated due to flow contracting and vortex stretching effects downstream of the channel junction. These are important factors of bed scouring and decreasing just downstream from tributary entry.

The application of the conventional flow model with the simplified helical flow to complex flows with a confluence is unrecommended because the relationship between depth averaged and bed surface velocity vectors differ considerably according to location, as shown in Figure 3. The present model can capture the above complex characteristics of flows with a channel confluence. The separation zones of the computation are larger than that of the experiment. 3D turbulence model by Hang *et al.* (2002) gave better results in the separation zone. This result indicates that the fluid mixing effect in the present model is still-inadequate in the separation zone. The further considerations to evaluate the fluid mixing effect by turbulence and vertical



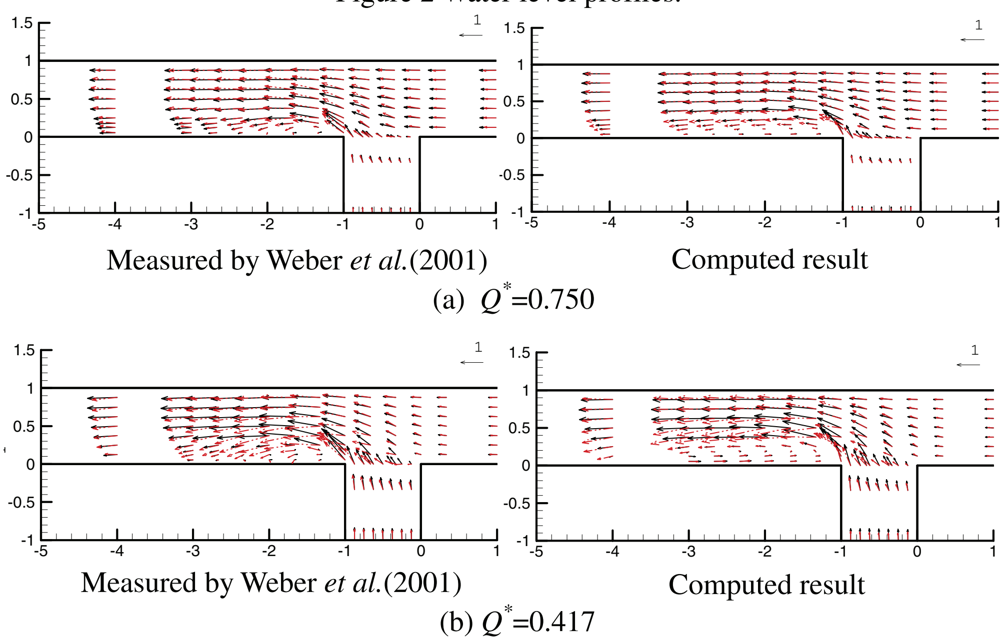
Measured by Weber *et al.*(2001) Computed result

(a) Q^*=0.750

Measured by Weber *et al.*(2001) Computed result

(b) Q^*=0.417

Figure 2 Water level profiles.



Measured by Weber *et al.*(2001) Computed result

(a) Q^*=0.750

Measured by Weber *et al.*(2001) Computed result

(b) Q^*=0.417

Figure 3 Depth averaged (solid black) and bed surface (dashed red) vectors.

velocity distribution in Eq.(9) and the diffusion effect of bed surface velocity by vertical velocity in Eq.(12) are required for the computation in the separation zone. However, the flow characteristics inducing bed scouring out of the separation zone are well-explained by the present model as well as full 3D model results (Hang *et al.*, 2002). The above indicates the present model is expected to be a practical flow model for bed variation computations.

CONCLUSION

This study proposes a new methodology of computing a 3D turbulence flow in channel confluences by depth-integrated equations of shallow water equations, a turbulence energy transport equation and horizontal vorticity equations. The important characteristic of the present model is that vertical velocity distributions are evaluated without the assumption of the hydrostatic pressure distribution. The complex flow characteristics inducing bed scouring around tributary entry are well-explained by the present model. The present model is expected to be a practical flow model for bed variation computations.

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