

# DEVELOPMENT OF ONE-DIMENSIONAL BED VARIATION ANALYSIS METHODS FOCUSED ON THE MECHANISM OF SEDIMENT TRANSPORT IN STONY-BED RIVERS

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## ABSTRACT

In a stony-bed river with wide grain-size distribution, boulders that resist movement during a flood act as strong fluid-resistance element. In addition, sand and gravel often remain in the bed irregularities formed by large stones. Therefore, a stony-bed river will remain in a static equilibrium condition, with no sediment discharge, when the river bed surface grain size distribution corresponding to tractive force of the flow are reached. However, conventional riverbed variation analysis does not account for this essential mechanism in stony-bed rivers. In this study, the authors developed a new method of one-dimensional (1-D) bed variation analysis for stony-bed rivers with well-graded sediment by considering the riverbed's stability mechanism created by large stones. We verified the applicability of the model with data from field experimentation carried out in the Joganji River.

*Keywords:* stony-bed river, well-graded sediment distribution, field experiment, bed variation analysis, boulder, static equilibrium

## 1. INTRODUCTION

Much research has been conducted to elucidate the mechanism of bed variation in stony-bed rivers—rivers in which the bed material is comprised of stone and gravel having a large grain-size distribution (e.g., Thorne, C.R. et al. 1987). However, most results obtained pertain to riverbeds comprised of gravel having particle sizes ranging from 0.2 to 7.5 cm; factors such as the difficulties of measurement in swiftly moving flows have impeded adequate research on the mechanism of bed variation in stony-bed rivers containing large stones (i.e., >30 cm).

The authors (Kuroda and Fukuoka et al. 2005; Fukuoka et al. 2006) have previously conducted large-scale field experimentation in the Joganji River to elucidate the mechanism of bed variation in stony-bed rivers. In a stony-bed river, bed scouring caused by flood flows exposes boulders and other large-diameter bed material, the immovability of which acts as a large resistance element against flood flows. Furthermore, the shielding effect of these large, immovable stones causes small- and medium-diameter gravel to collect in the wake of these obstacles. Consequently, the formation in a stony-bed river of a bed shape and bed material size distribution suited to the flow will achieve static equilibrium in which neither stone nor gravel are transported. This is an important mechanism of bed variation in stony-bed rivers.

Conventional techniques for analyzing bed variation in stony-bed rivers in general consist of bed load equation with mixtures and continuity equations for sediment and grain

sizes (Hirano, 1971). The equations of Ashida and Michiue (1972) are widely used to calculate bed load equation for sediment mixtures. To calculate critical tractive force for sediment mixtures in those equations, the equation of Egiazaroff (1965) as modified by Ashida and Michiue (1972) are often used. However, Fukuoka et al. (2006) have compared their results with critical tractive force for sediment mixtures calculated with the modified Egiazaroff equation and found that in an actual bed that has achieved static equilibrium, the modified Egiazaroff equation indicates that gravel would be transported. This showed that the modified Egiazaroff equation does not adequately account for the aforementioned shielding effect of large stones and so is difficult to apply to stony-bed rivers.

In a stony-bed river, stones and gravels are moved violently during flooding, readily leading to bank erosion and bed scouring near revetments. Countermeasures to these problems include underpinning for existing revetments and the construction of new revetments at locations of bank erosion, but such measures have been ineffective because of the inability to predict the location and scale of scouring and erosion. In addition, dams and other lateral structure have blocked the downstream transport of large-diameter bed material, while materials of specific sizes have been removed from riverbeds for use in construction, etc., thereby altering grain-size distributions. The loss of large-diameter materials has reduced the ability of such riverbeds to resist flood flows, which could lead to even greater bed scouring. To determine which bed material populations contribute to the formation of stable beds, it is important to establish a highly accurate technique for analyzing bed variation in stony-bed rivers.

The authors have established a one-dimensional bed variation analysis method that focuses on the mechanism of stone and gravel transport. This technique takes an integrated view of the temporal changes in water surface profile and bed variation and how these quantities vary according to discharge. In particular, the determination of bed variation accounts for the static equilibrium of bed materials, which is an important mechanism of bed variation in stony-bed rivers. This technique was validated by applying it to large-scale field experimentation in the Joganji River and comparing the resulting calculations to the observed data. This paper also discusses the differences in approach between the authors' technique and existing techniques and enumerates the problems with the latter.

## **2. A NEW METHOD FOR THE ONE-DIMENSIONAL ANALYSIS OF BED VARIATION**

This section describes the new technique that the authors have formulated for the analysis of bed variation in stony-bed rivers—a method that takes into account the mechanism of stone and gravel transport. The underlying philosophy of this method is also discussed.

In a stony-bed river containing large stones, these stones are exposed in the course of erosion and thereby create resistance to the fluid field. In addition, the stones' shielding effect causes gravel to collect around the stones. Because large stones thus contribute greatly to bed stability, methods for analysis must adequately account for this effect. Believing that the effect of large stones should be incorporated as a form of bed irregularities, and rather than assuming constant bed height in the computational grid as conventional bed variation analysis techniques do, the authors devised a method of calculating the height of each size group of materials on the riverbed surface. To incorporate this method, the authors created equations in which a transported-gravel layer (i.e., movable particle layer) is completely separated from the layer of nonmovable stones and gravel (i.e., the bed) (see Figure 1) and sediment discharge and bed variation are calculated using quantities picked up from the bed and deposited to the bed. The flow of calculations is shown below.

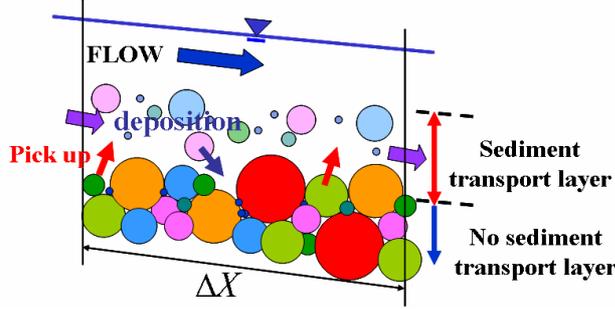


Figure 1 Concept of the new analysis method

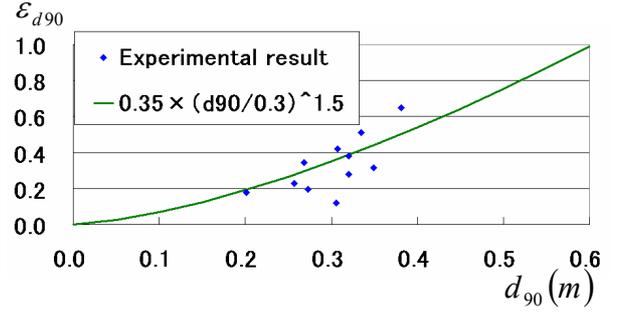


Figure 3 Relationships between  $\varepsilon_{d90}$  and  $d_{90}$

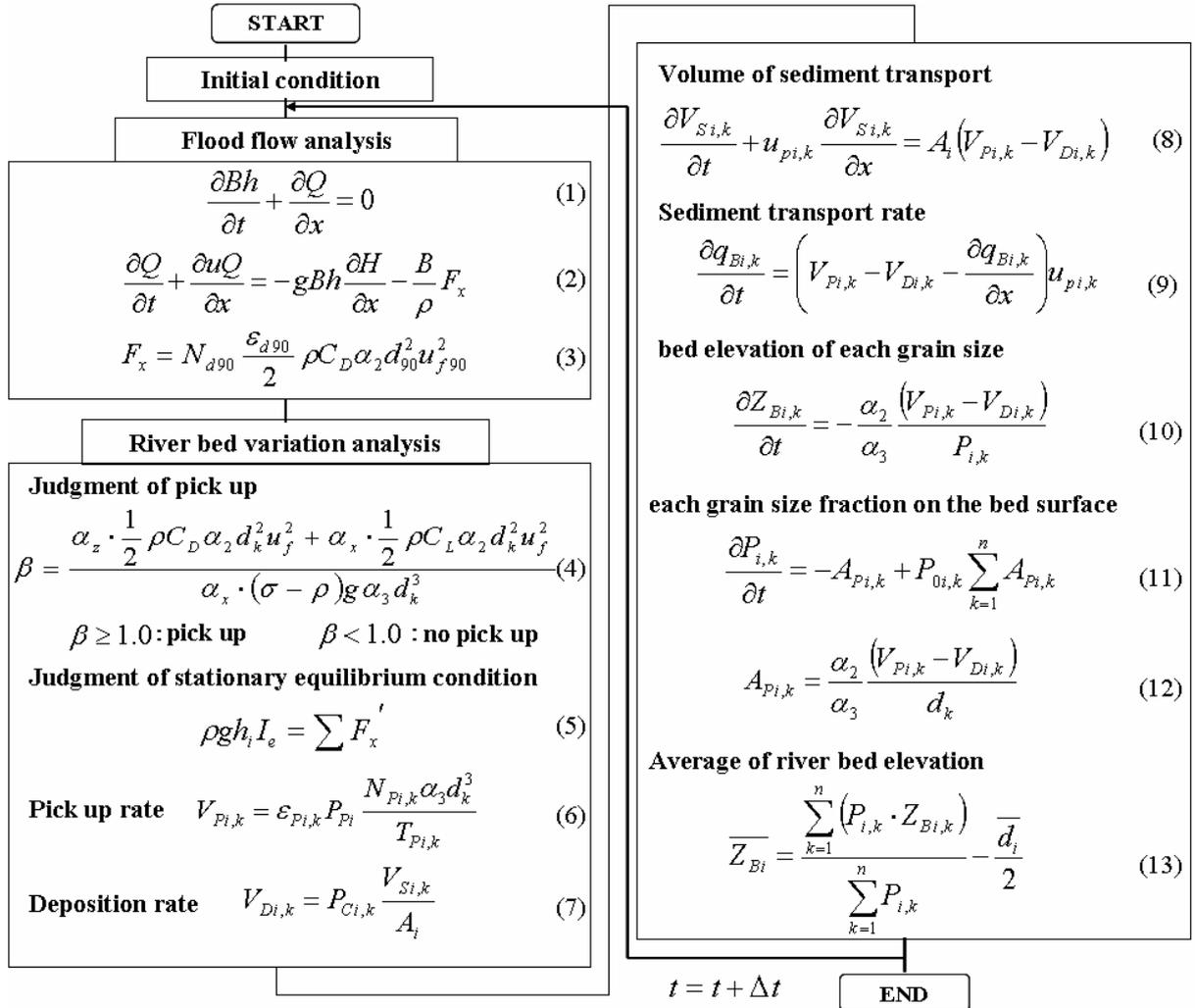


Figure 2 Process of the new bed variation analysis

Figure 2 shows the flow of calculations in the authors' bed variation analysis technique. The flow field is calculated with equations (1) and (2). Although the bed resistance term generally used in the equation of motion is Manning's roughness coefficient or some other representation of frictional resistance, in a river with stones and other large-diameter bed materials, the form resistance of these materials is predominant. Therefore the authors, viewing riverbed materials of size  $d_{80}$  or greater as contributing to resistance (Fukuoka et al., 2006), determined resistance with equation (3), which uses  $d_{90}$ , the intermediary between  $d_{80}$  and the maximum diameter.  $N_{d90}$  is the number of particles contributing to bed resistance per unit area. As particles of diameter  $d_{80}$  or greater contribute to resistance, this number is 20%

of the unit area. The number of  $d_{90}$  in 20% of the unit area is calculated with the following equation.

$$N_{d_{90}} = \frac{0.2}{\alpha_2 d_{90}^2} \quad (14)$$

where  $\alpha_2, \alpha_3 = 2\text{-D}$  and  $3\text{-D}$  shape factors of the particles,  $C_D = \text{drag coefficient}$ . Since actual gravel particles have an irregular shape, not a spherical one, and since the particles are assumed to be half-buried, a value of 1.0 was used for  $C_D$  in equation (3), which is also based on the values for submerged roughness drag coefficient observed by Uchida and Fukuoka et al. (2001). In equation (3),  $\varepsilon_{d_{90}}$  is the shielding coefficient of  $d_{90}$  and the product of  $\varepsilon_{d_{90}} \alpha_2 d_{90}^2$  is the projectional area of  $d_{90}$  in the flow direction. Figure 3 shows the correlation between the experimentally determined (Kuroda and Fukuoka et al., 2005)  $d_{90}$  and  $\varepsilon_{d_{90}}$ . Based on this, the equation for calculating  $\varepsilon_{d_{90}}$  is as follows.

$$\varepsilon_{d_{90}} = 0.35 \left( \frac{d_{90}}{0.3} \right)^{1.5} \quad (15)$$

Also,  $u_{f90}$  is the velocity acting on  $d_{90}$  and is determined with a logarithmic velocity distribution equation.



(a) Concept of the pick up particle (b) Calculation method of  $\gamma_{k,k'}$   
Figure 4 Concept of the pick up particle and calculation method of  $\gamma_{k,k'}$

The first step in bed variation analysis is to determine whether or not each particle size group is picked up according to balance-of-moment equation (equation (4)). An overview of the process of pick up judgment is shown in Figure 4(a). For each particle whose pick up is to be determined, a stationary particle (of diameter  $d_{80}$ ) is established below angle  $\theta_k$ , and equation (4) is calculated with its point of contact as the fulcrum. Here,  $C_L$  is the lift coefficient, which was assigned a value of 0.2 based on the work of Fukuoka and Watanabe et al. (2005). In addition,  $\alpha_x$  is the distance in direction  $x$  from the contact point to the center of the separated particle,  $\alpha_z$  the distance in direction  $z$  from the contact point to the point where fluid force acts, and  $u_f$  the velocity acting on the particle, which are calculated according to logarithmic law using the average bed (discussed below) as the standard surface. The subscript  $k$  indicates the particle diameter group. The point where fluid force acts was defined as the  $0.8d_k$  portion of a separating particle, as shown in Figure 4(a). The angle  $\theta_k$  was calculated as shown below using diameter  $d_{80}$ .

$$\theta_k = 90.0 - 45.0 \exp \left\{ \left( Z_{Bi,k} - Z_{d_{80}i} \right) \frac{d_{80i}}{d_k} \right\} \quad (16)$$

where  $Z_{Bi,k} = \text{height of diameter } d_k$ ,  $Z_{d_{80}i} = \text{height of } d_{80}$ , and subscript  $i$  indicates the computational grid number. This equation represents the relationship between  $d_{80}$  and the height-diameter ratio of the picked up particles. For instance, when the diameter is less than  $d_{80}$  and height is low, then angle  $\theta_k$  is greater than  $45^\circ$ , indicating pick up is less likely.

Equation (16) shows that the shielding effect of large-diameter particles reduces the pick up rate of smaller-diameter particles and is also relevant to pick up time, which is discussed in greater detail below. This is an important indicator of pick up, although because sufficient verification has not been conducted, additional study is necessary.

Next we explain the method of determining bed stability. Equation (5) represents the conditions for bed stability. The left side is the downstream-direction component of fluid mass, while  $F'_x$  is the resistance from those bed particles of size  $d_{80}$  or greater determined by equation (4) to resist transport. We assume that when all particles of size  $d_{80}$  or greater resist transport, it means that all drag from the fluid field is absorbed by those particles, and fluid force does not act on the surrounding particles. In the calculations, pick up analysis is conducted on particles of size  $d_{80}$  and  $d_{90}$ ; when both are found to resist pick up (i.e., when sizes  $d_{80}$  and greater are stable), it is assumed that all fluid drag is absorbed by particles of size  $d_{80}$  and greater, with no drag acting on the other particles, and so the quantity of pick up from the bed is deemed to be zero.

After the process outlined up to this point, equation (6) is used to determine the quantity of pick up, i.e., the quantity of particles found to pick up from the bed. Here,  $V_{Pi,k}$  is the pick up quantity per unit time and unit area,  $N_{Pi,k}$  is the number of particles of each diameter group in the bed, and  $T_{Pi,k}$  is the time required for pick up from the bed—using the equation of motion for particles, this is defined as the time required for a particle to be transported to the top of a stationary particle, as shown in Figure 4(a). However, even if pick up judgment shows particles of diameter  $k$  to be picked up, it is possible that not all particles of diameter  $k$  on the bed surface will pick up. For instance, some such particles may be shielded by larger particles, or irregularities in the bed surface may prevent forces from acting uniformly on all particles of that diameter. Consequently, rather than picking up all particles of diameter  $k$  on the surface, the number of picking up particles must be controlled. In equation (6),  $\varepsilon_{Pi,k}$  is a coefficient that accounts for the shielding effect of particles larger than  $d_k$ , with no shielding effect on particles larger than  $d_{80}$  ( $\varepsilon_{Pi,k} = 1.0$ ), while the shielding effect on particles smaller than  $d_{80}$  is calculated with the following equation.

$$\varepsilon_{Pi,k} = 1.0 - \sum_{k'=1}^{k-1} (\gamma_{k,k'} \cdot P_{i,k'}) \quad (17)$$

$P_{i,k}$  is the bed surface ratio of particles of diameter  $d_k$ , and  $\gamma_{k,k'}$  is defined as follows in relation to the relative heights of particles  $k$  and  $k'$ . As shown in (1) in Figure 4(b), the height of the bottom of particle  $k$  is compared to the height of the top of particle  $k'$ , and if the bottom of  $k$  is higher, then the value of  $\gamma_{k,k'}$  is zero, i.e., particle  $k'$  has no shielding effect on particle  $k$ . When, as in (2) in Figure 4(b), the height of the bottom of particle  $k$  is compared to the center of particle  $k'$ , if the bottom of particle  $k$  is lower then the value of  $\gamma_{k,k'}$  is 1. For particles between (1) and (2) the value of  $\gamma_{k,k'}$  is determined linearly as in Figure 4(b).  $P_{Pi}$  is the ratio between the fluid field force acting on the bed and the sum total of drag acting on the particle, and is calculated as follows:

$$P_{Pi} = \frac{\rho g h_i I_e}{\sum_{k=1}^n \left( \frac{N_{Pi,k}}{2} \rho C_D \alpha_2 d_k^2 u_f^2 \right)} \quad (18)$$

Next, we explain how deposition rate on the bed is calculated. The amount of deposition on the bed is calculated with equation(3), in which  $V_{Di,k}$  is bed deposition quantity per unit time and unit area,  $P_{Ci,k}$  is the collision rate (i.e., rest rate), and  $V_{Si,k}$  is the sediment volume within the computational grid, which is calculated with equation (8). In addition,  $A_i$  is the computational grid area ( $A_i = \Delta x_i \times B_i$ ) and  $u_{pi,k}$  in equation (8) is particle velocity in the downstream direction.

The rest ratio is an important variable in the equation for bed deposition. The author's method includes calculating 5-second saltation using the particle equation of motion, then determining the bed particle collision ratio, which is assumed to be the rest ratio (Figure 5). An actual riverbed is comprised of a broad range of particle size populations and has considerable irregularities, with particles conceivably being stopped by depressions and collisions with large particles. However, the authors' method does not yet account for this. The method for determining the rest ratio must be improved through the detailed examination of field data. Particle velocity ( $u_{pi,k}$ ) is determined by dividing the calculated time (5 seconds) into the particle's downstream movement distance as determined by the saltation time calculations. In addition, sediment discharge is calculated not with the sediment-transport equation but in a way that accounts for the balance between pick up rate and deposition rate.

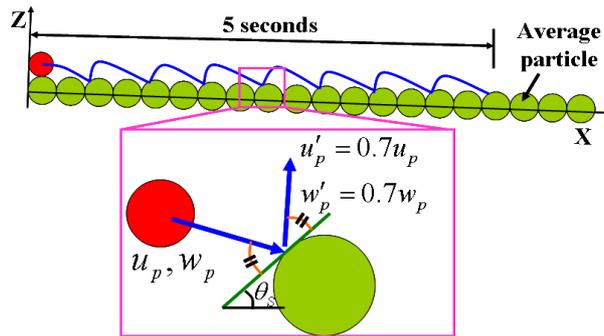


Figure 5 Concept of the saltation analysis

Next, let us see how the authors calculate the height of each particle size group (defined as the height of the particle tip), bed surface ratio of  $d_k$  particles and average bed height. The height of each particle size group moment by moment is calculated using each size group's pick up quantity from the bed, deposition quantity to the bed, and the bed surface ratio of  $d_k$  particles with equation (10). As seen in equations (11) and (12), the bed surface ratio of each particle size group is calculated using each size group's pick up and deposition quantity. Here,  $P_{0i,k}$  is the subsurface particle diameter ratio. Average bed height is needed to calculate the flow field and is calculated as shown in equation (13). The first term on the right side of equation (13) is the average particle height in the computational grid, and the second term on the right is radius of the average-diameter particle. Thus, average bed height is defined as average particle height less the radius of the average-diameter particle.

### 3. VALIDATION THROUGH APPLICATION TO 2004 FIELD EXPERIMENTS

#### 3.1 Conditions of Calculations

The authors tested their bed variation analysis method by applying it to the results of field experiments conducted in the Joganji River in 2004. In the experiment, a steady flow was released through a 170-meter-long linear waterway. Water level, discharge, bed height, and bed surface particle size distribution were measured. Figure 6 shows the profile of the waterway's bed height, water level, and waterway width. A rapidly-flowing segment formed in the waterway's middle, where width also narrowed. This narrowing occurred because prior to the start of the experiment, bed materials that were larger than materials elsewhere in the waterway were in existence there. Consequently, the initial surface and subsurface particle size distribution used for this swiftly-flowing section in the calculations was larger than the slowly-flowing section. The grain size distribution was based on the dimensionless particle

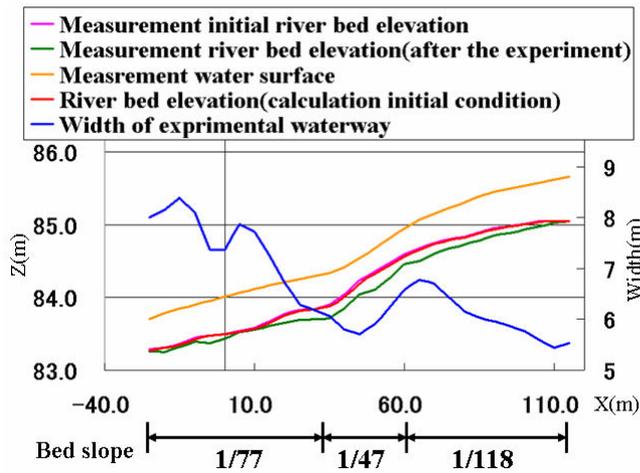


Figure 6 Longitudinal distribution of water elevation, bed elevation and waterway width

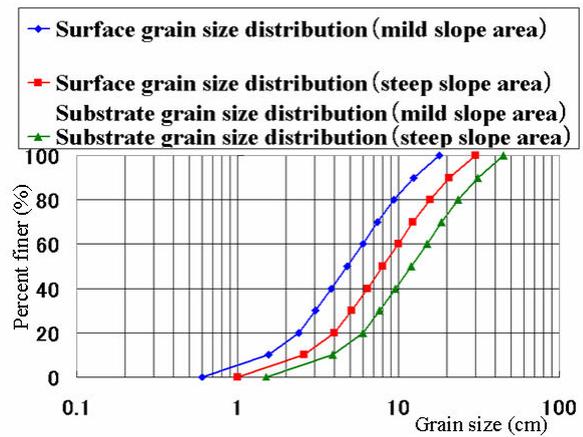
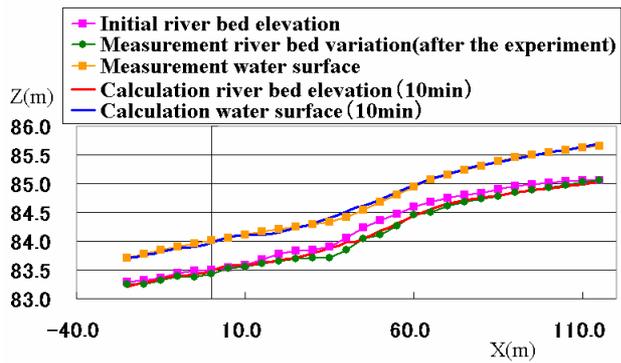


Figure 7 Initial condition of grain size distributions

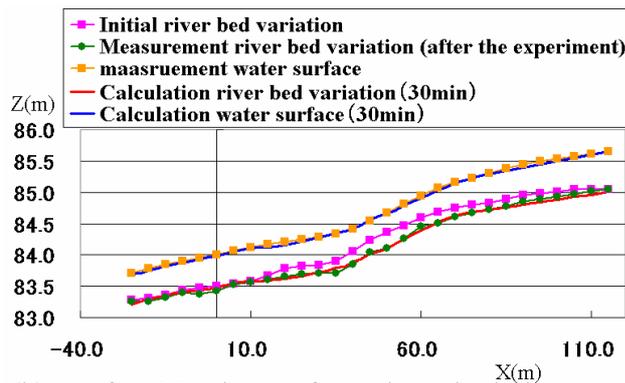
size distribution determined by Kuroda and Fukuoka et al. (2005). Seven particle sizes were used in the calculations—35 cm, 27 cm, 20 cm, 12 cm, 8 cm, 5 cm, and 2.5 cm. The initial bed height used in the calculations was that from the experiment (Figure 6), and the bed variation calculations were run until bed height stabilized throughout the entire waterway.

### 3.2 Results of Calculation

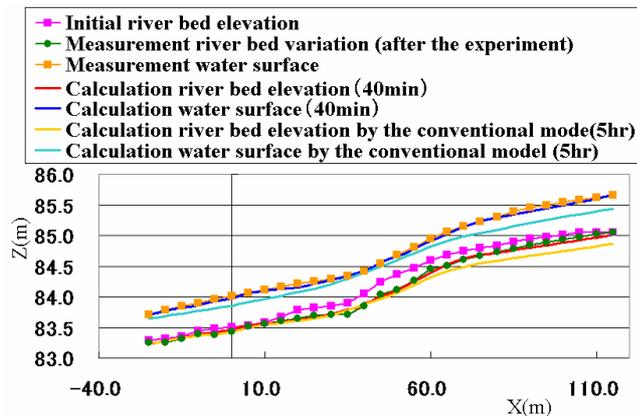
Figure 8 compares the calculated and experimentally obtained water level and bed height profiles over time; Figure 9, bed transport rate over time of each bed material size. In the experiment, static equilibrium was achieved approximately 24 minutes after the flow of water began. In the calculations, bed material transport had nearly stopped—excepting in the downstream section—after 30 minutes and within 40 minutes had completely stopped throughout the waterway, achieving static equilibrium. Looking at the results for water level and bed height after 40 minutes (Figure 8(c)), we see that the calculations accurately reproduce the experimentally obtained water surface profile and bed height. Figure 8(c) also shows the results of calculations performed with a conventional technique (described below) simulating conditions after 5 hours of flow. In these conventional simulations, large-diameter bed particles were unmoved while small-diameter particles moved continuously, resulting in extremely slow bed variation and a bed height that continued to drop even after 5 hours. In short, results differed greatly from the experimentally observed values for both water level and bed height. The data in Figure 9 for the transport of each bed material size indicate extensive transport of 8-cm, 12-cm, and other cobble-class material, whereas transport of 35-cm, 27-cm, and other large-sized rocks was limited. The low amount of transport of large rocks is probably due to the low pick up amount and the slow transport velocity, while the shielding effect of the large-sized bed material—which was accounted for in the calculations—reduced the pick up rate of 2.5-cm and other small-sized bed material. Figure 10, a comparison of experimental and calculated results for each particle size ( $d_{20}$ ,  $d_{60}$ ,  $d_{80}$ , and  $d_{90}$ ) when static equilibrium had been reached after 40 minutes of flow, shows that except for the upstream reaches, the calculations generally agree with the experimental results. The particularly good reproducibility (i.e., predictive accuracy) for  $d_{80}$  and  $d_{90}$  are likely a result of the static equilibrium achieved through the entire waterway by that point. Thus, the authors' technique is generally capable of explaining water surface profile, quantitative bed variation, and bed material size distribution in stony-bed rivers with a wide range of bed material sizes.



(a) After 10 minutes from the calculation start

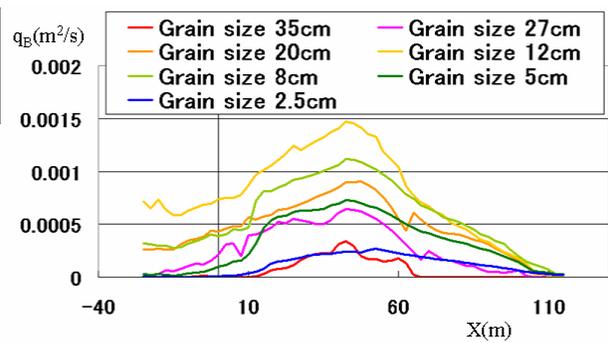


(b) After 30 minutes from the calculation start

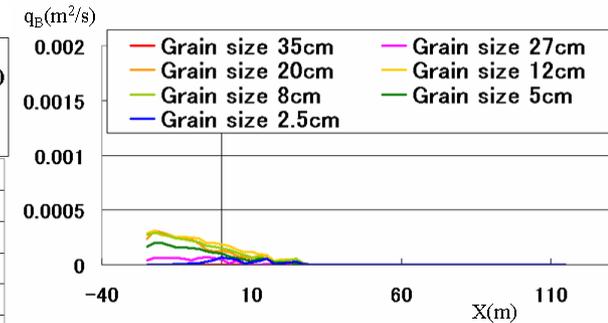


(c) After 40 minutes from the calculation start

Figure 8 Comparison between the calculated result and the measured result (water surface and river bed elevation)



(a) After 10 minutes from the calculation start



(b) After 30 minutes from the calculation start

Figure 9 Calculated result of Sediment transport rate

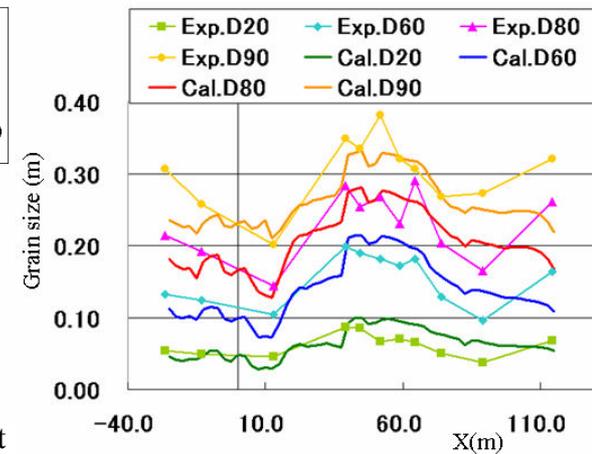


Figure 10 Comparison of the calculated result and the measured result (D90, D80, D60, D20)

### 3.3 Differences in Fundamental Approaches between the Authors' Technique and Conventional Bed Variation Analysis Techniques

Applying conventional bed variation analysis to field experiments produces results that differ greatly from the observed data. This section compares the fundamental approaches of conventional analysis and the authors' technique and discusses problems with the former. In general, conventional analysis uses sediment discharge equations, e.g., those of Ashida and Michiue (1972), to determine the transport of each particle size. Ashida and Michiue derive their bed load equation for sediment mixtures using critical tractive force

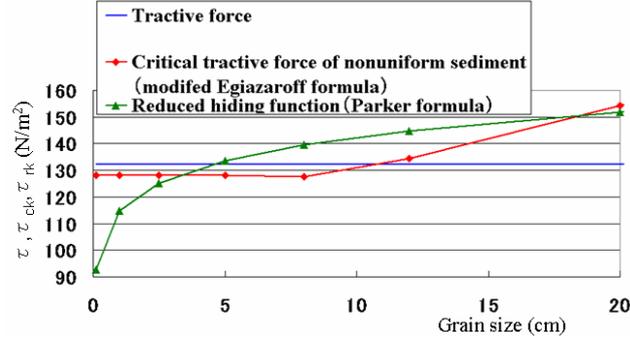


Figure 11 Comparison of the tractive force and the critical tractive force

With sediment mixtures and relative share of each particle size in the bed materials in an equilibrium sediment discharge equation determined for uniform and fine sediment. However, the sort of stony-bed rivers shown in Figure 1 contain large, greatly exposed, immovable rocks, which prevent the sort of continuous bed material movement assumed by Ashida and Michiue and instead result in the discontinuous movement of all bed material sizes. The authors' method accounts for such discontinuous movement and predicts the volume of sediment discharge from bed material pick up volume and deposition volume, as in equations (8) and (9). Furthermore, bed load equation with mixtures derived by Ashida and Michiue uses critical tractive force with mixtures—an important indicator—based on a modified theory of Egiazaroff (1965). The Egiazaroff equation is:

$$\tau_{*ck} = \frac{0.1}{\left\{ \log_{10} \left( 19d_k / d_{avg} \right) \right\}^2} \quad (19)$$

In which critical particle transport velocity is calculated with a balance relation equation assuming sliding motion. However, the manner of particle separation in a stony-bed river is not sliding but, because of the extensive irregularities in the bed surface as shown in Figure 1, is more likely explained by rolling motion in which one particle rolls over another. This is why the authors make pick up judgment of particles in the manner seen in equation (4). In Figure 11, tractive force at the point  $X = 40m$  after bed stability had been achieved as calculated in Figure 8 is compared with the critical tractive force for each particle size as calculated with the modified Egiazaroff equation. The modified Egiazaroff equation predicts that, in a state in which the authors' analysis predicts bed stability having been achieved, particles smaller than 10 cm would be subjected to a tractive force exceeding the critical tractive force and would therefore be transported.

Figure 11 also shows standard tractive force according to Parker (1990) (below which only extremely little sediment discharge is believed to occur). Parker's theory assumes that particles smaller than 5 cm are transported and so does capture the phenomenon to some degree. However, because of the issue of porosity (discussed below) in conventional bed variation analysis techniques, even if the concept of standard tractive force is incorporated, the results will likely show a similar tendency as the Egiazaroff calculation results in Figure 8(c).

Hirano's equations (Hirano, 1971) are widely used as one-dimensional continuity equations for mixed sand-and-gravel beds (only the equation for bed lowering is shown).

$$\frac{\partial Z_B}{\partial t} = -\frac{1}{B(1-\lambda_0)} \frac{\partial(q_B \cdot B)}{\partial X} + \frac{a}{1-\lambda_0} \frac{\partial \lambda}{\partial t} \quad (20)$$

The second term on the right side of the equation does account for temporal change in porosity. Because of the difficulty of calculating temporal change in porosity, however,

porosity is usually treated as constant and the term is disregarded. Yet the large stones, etc., in a stony-bed river result in considerable surface irregularity and large voids around the rocks. Calculations that treat the voids as constant indicate large rocks, which in reality would not reduce in height, as lowering as the surrounding gravel is transported, yielding results suggesting significant bed variation. By calculating height separately for each bed particle size, the authors' method accounts for porosity arising from bed irregularity, which is an important mechanism determining bed variation in stony-bed rivers.

#### 4. CONCLUSIONS AND ISSUES FOR FUTURE STUDY

(1) The authors have devised a new one-dimensional bed variation analysis method for stony-bed rivers and validated this method by using it to reproduce results obtained in field experimentation in the Joganji River. This validation showed a high degree of reproducibility of important determinants of riverbed variation—the temporal and spatial distribution of water level profile, bed variation, and bed particle size distribution—and the method reproduced the bed variation mechanism of stony-bed rivers to a degree not possible with conventional analysis.

(2) Further investigation is needed to determine the validity and range of applicability of this method in determining, for stony-bed rivers, the shielding effect on small bed particles of large bed particles, an important mechanism in this method.

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