Bed variation analysis using the sediment transport formula considering the effect of river width and cross-sectional form in the Ishikari River mouth

SEIJI OKAMURA[1], KAZUNORI OKABE[2] & SHOJI FUKUOKA[3]

1 IDEA Consultants, Inc., 3-15-1, Komazawa, Setagaya-ku, Tokyo, Japan
seijiokamura5@gmail.com

2 Hokkaido Bureau, Ministry of Land, Infrastructure, Transport and Tourism, 2-1-3, Kasumigaseki, Chiyoda-ku, Tokyo, Japan

3 Research and Development Initiative, Chuo University, 1-13-31214, Kasuga, Bunkyo-ku, Tokyo, Japan

Abstract For river management, it is important to estimate the sediment discharge rate and the degree of bed variations during floods. Many of the previous sediment transport formulae have been proposed based on results by experimental channels. Therefore, they cannot estimate well the sediment discharge rate in rivers because of the complicated channel shapes and bed forms. It is necessary to calculate the sediment discharge rate appropriately to improve the accuracy of bed variation analysis. Fukuoka (2010) thought that stable cross-sectional scales of alluvial rivers (such as width and depth) are determined by physical quantities which indicate characteristics of basins (such as discharge, river bed slope and river bed material). He then derived formulae between dimensionless quantities of width, depth and discharge using field observed data by dimensional analysis. Based on the above analysis, he also derived the bed load formula considering the effect of river width and cross-sectional form using field observed data. In this study, we develop a bed variation analysis with the Fukuoka's bed load formula to calculate the 1981 flood of the Ishikari River that caused large bed scouring at the river mouth. We compare calculated results between the bed load formula of Fukuoka (2010) and the previous formulae of Ashida & Michiue (1972) and Sato *et al.* (1958) for the amount of bed load discharge rates during the flood and the bed forms after the flood. As a result, the cross-sectional bed forms at meandering channel are well reproduced when we use the bed load formulae of Fukuoka (2010) and Sato *et al.* (1958). This indicates that these bed load formulae can calculate the amount of bed load discharge rate appropriately in the case of the Ishikari River mouth.

Key words Fukuoka's bed load formula; bed variation analysis; Fukuoka's equation; river mouth

INTRODUCTION

For river management, it is important to estimate the sediment discharge rate and the degree of bed variations during floods. But it is difficult to measure temporal variations of bed elevations and sediment discharge rate during floods. Many of the previous sediment transport formulae have been proposed based on results by simplified experimental channels. Therefore, they cannot estimate well the sediment discharge rate in rivers because of the complicated channel shapes and bed forms.

Fukuoka (2010) thought that stable cross-sectional scales of alluvial rivers (such as width and depth) are determined by physical quantities which indicate characteristics of basins (such as discharge, river bed slope and river bed material). Then, he derived formulae between dimensionless quantities of width, depth and discharge by the dimensional analysis using field data observed in natural rivers and experimental channels whose width vary with bank erosion (equations (1) and (2)).

$$\frac{B}{d_r} = 4.25\left(\frac{Q_d}{\sqrt{gId_r^{\,5}}}\right)^{0.40} \tag{1}$$

$$\frac{h}{d_r} = 0.13\left(\frac{Q_d}{\sqrt{gId_r^{\,5}}}\right)^{0.38} \tag{2}$$

where B = surface width, h = section-averaged depth, Q_d = dominant discharge, g = gravitational acceleration, I = energy gradient, and d_r = representative grain diameter.

Fukuoka (2010) thought that sediment discharge rate should have a relationship with dimensionless discharge because sediment transports and bed variations during floods occur in the stable river channels which have been formed by dominant physical quantities of basins. Then, he indicated that dimensionless bed load discharge rate is determined by the product of dimensionless discharge and energy gradient by using field observed data, as shown in Fig. 1 and equation (3), using field observation data of Nakato *et al.* (1977, 1981), USGS (1989)and Fukuoka (2010). Data of bed load discharge rates in Fig. 1 are observed at stable river sections which satisfy the relationships in equations (1) and (2).

$$\frac{Q_B}{\sqrt{sgId_r^{\,5}}} = 0.02\left(\frac{Q}{\sqrt{gId_r^{\,5}}}I\left(1-\frac{I_c}{I}\right)\right) \tag{3}$$

where Q_B = bed load discharge rate of full cross-section, Q = observed discharge (when bed load discharge rates were measured), s = specific gravity of sediment in water, and I_c = critical gradient for sediment movement.

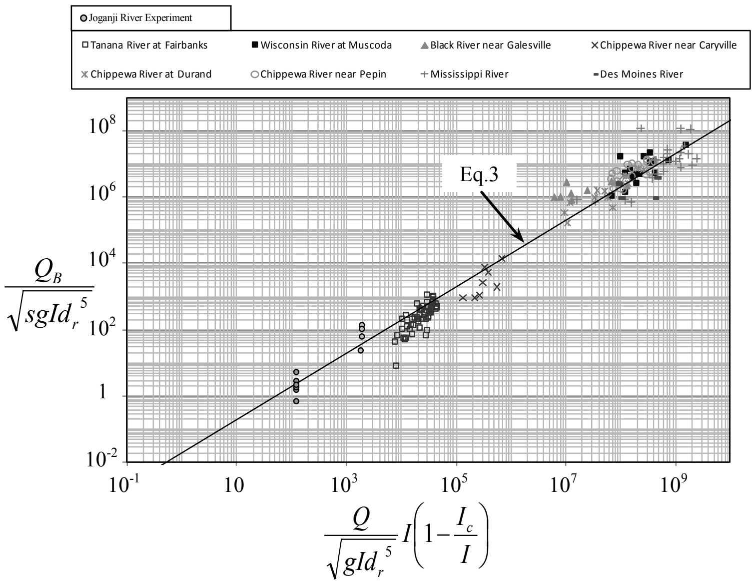

Fig. 1 Relationship between dimensionless bed load discharge rate and the product of dimensionless discharge and gradient of full cross-section.

In this study, we develop a numerical analysis of unsteady flow and bed variation with the Fukuoka's bed load formula to calculate the 1981 flood of the Ishikari River which caused a large bed scouring at the river mouth. We compare calculated results between the bed load formula of Fukuoka (2010) and the previous formulae of Ashida & Michiue (1972) and Sato *et al.* (1958) for the amount of bed load discharge rates during the flood and the bed forms after the flood. Then we investigate the effect of bed load formula for the accuracy of bed variation analysis.

AUGUST 1981 FLOOD IN THE ISHIKARI RIVER MOUTH AND THE PREVIOUS STUDIES

Figure 2 shows a discharge hydrograph of August 1981 flood of the Ishikari River mouth. The peak discharge of the flood exceeded the design discharge of the time. During the flood, temporal data of water levels were measured at many observation points in the reach 15 km upstream from the river mouth, as shown in Fig. 3.

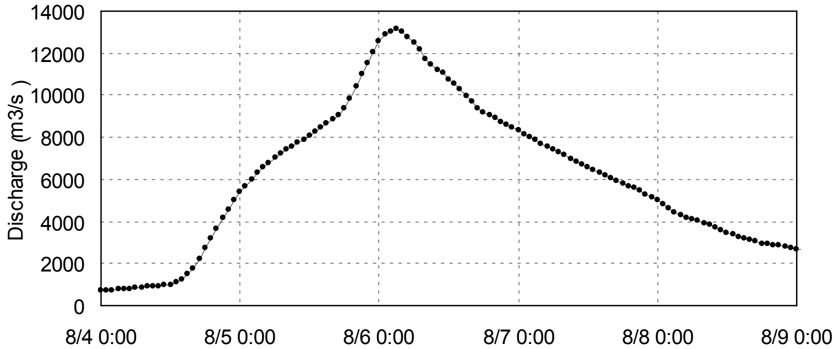

Fig. 2 Discharge hydrograph of August 1981 flood of the Ishikari River mouth.

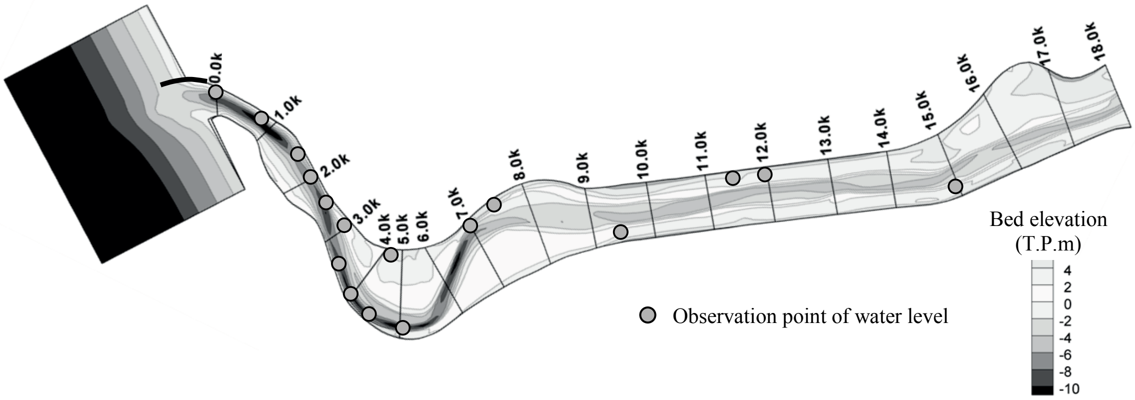

Fig. 3 Planform of the Ishikari River mouth and observation points of 1981 flood.

There has been considerable research on the bed variation of the 1981 flood in the Ishikari River Mouth. Mori *et al.* (1985) developed 3D flow and 2D bed variation analysis to the meandering compound channel of the Ishikari River mouth under constant discharge conditions. He indicated that scouring areas shift to the centre of the main channel under the influence of inflow from the flood plain. Shimizu *et al.* (1986) focused on the role of suspended load to estimate the bed variation by the flood. He developed 1D quasi-steady flow and bed variation analysis considering the concentration of suspended load whose vertical distribution is assumed to be an exponential curve. Inoue *et al.* (2004) extended the above analysis to 2D quasi-steady flow and bed variation analysis considering the effect of secondary flow, where, the flux of suspended load is calculated by concentration of suspended load and horizontal velocity whose vertical distributions are assumed to be exponential and polynomial curves, respectively.

Okamura *et al.* (2010) elucidated the bed variation during the flood by the quasi-3D unsteady flood flow and 2D bed variation analysis using observed temporal changes in water surface profiles of the flood. This computational method is based on the idea that the influences of channel shape, bed resistance, vegetation, bed variation during flood, etc. are reflected in temporal changes in water surface profiles. In the analysis, the concentration of suspended load is calculated by 3D advection-diffusion equations where the vertical distributions of velocities are calculated by quasi-3D unsteady flow analysis. But a problem to be solved remains that the cross-sectional bed slope of calculation was steeper than that of observation. It is considered as one of the factors that the amount of lateral sediment discharge rate is not estimated properly.

In this study, we calculate the amount of bed load discharge rate using the bed load formula of Fukuoka (equation (5)) and the previous formulae of Ashida & Michiue (1972) and Sato *et al.* (1958). Then we investigate the effect of sediment transport formula for the accuracy of bed variation analysis.

THE ANALYSIS OF BED VARIATION DURING 1981 FLOOD IN THE ISHIKARI RIVER

Computational method

We develop the unsteady numerical analysis of 1981 flood flow and bed variation using observed temporal changes in water surface profiles. The analysis consists of quasi-3D unsteady flow analysis (Uchida & Fukuoka, 2009) and 2D bed variation analysis for graded sediment (Fukuoka, *et al.*, 1998), employing the general coordinate system. In order to calculate the suspended load affected by the secondary flow of meandering channel, the concentration of suspended load is calculated by 3D advection-diffusion equations. The pick up rate of suspended load from bed surface is calculated by the formula of Itakura & Kishi (1980).

Fukuoka's bed load formula

Equation (3) is a relationship between cross-sectional bed load discharge rate and discharge over a full cross-section. This equation is not applicable to a horizontal 2D bed variation analysis, because the analysis requires local bed load discharge. In this study, equation (3) is transformed to equation (4) for the 2D bed variation analysis. Equation (4) is a relationship between bed load discharge rate per unit width and discharge per unit width. This equation also corresponds with observed field data as shown in Fig. 4. These are same data with those of Fig. 1.

$$\frac{q_B}{\sqrt{sgId_r^{\ 3}}} = 0.02\left(\frac{q}{\sqrt{gId_r^{\ 3}}}I\left(1-\frac{I_c}{I}\right)\right) \tag{4}$$

where q_B = bed load discharge rate per unit width, and q = discharge per unit width. Equation (4) is the relational expression for d_r (representative grain diameter).

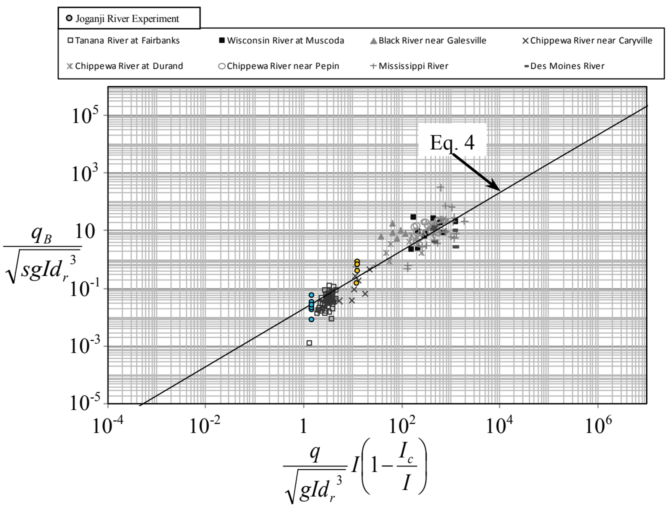

Fig. 4 Relationship between dimensionless bed load discharge rate per unit width and the product of dimensionless discharge per unit width and gradient.

In this study, in order to apply Fukuoka's bed load formula to graded sediment condition, we assume that equation (4) can be extended for d_k (arbitrary grain diameter), as shown in equation (5).

$$\frac{q_{Bk}}{\sqrt{sgId_k^{\ 3}}} = 0.02P_k\left(\frac{q}{\sqrt{gId_k^{\ 3}}}I\left(1-\frac{I_{ck}}{I}\right)\right) \tag{5}$$

where q_{Bk} = bed load discharge rate per unit width of d_k, and P_k = ratio of d_k on bed surface. Critical gradient for sediment movement I_{ck} is obtained by the follow equation:

$$I_{ck} = \tau_{*ck} \frac{sd_k}{h} \tag{6}$$

where τ_{*ck} = dimensionless critical shear stress. We use the value of τ_{*ck} = 0.05 in this study.

Fukuoka's bed load formula is based on the field data (shown in Figs 1 and 4). These data are observed in stable river sections which satisfy the relationship in equations (1) and (2). Therefore, before applying Fukuoka's bed load formula, we check the relationships between dimensionless width, depth and discharge of the section in question.

Figure 5 shows the relationships between dimensionless width, depth and discharge of section from 1 km to 15 km during the 1981 flood. In this section the channel shape is not uniform. As shown in Fig. 3, it is straight channel upstream of 9 km, wide compound meandering channel from 4 km to 9 km section and narrow single cross-section downstream of 3 km. The relationships between dimensionless width, depth and discharge correspond well with equations (1) and (2) in each section.

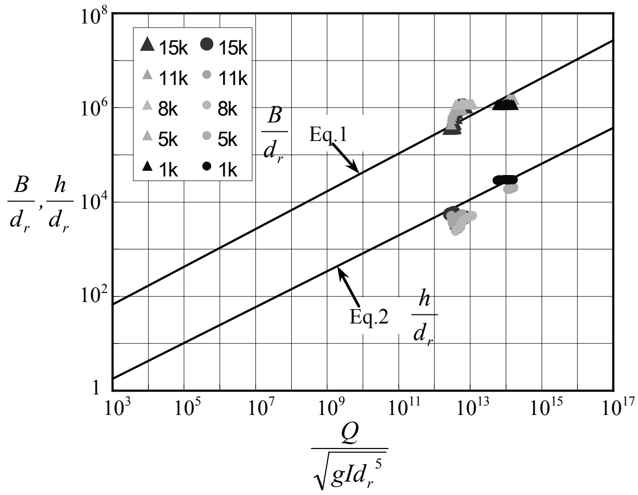

Fig. 5 Relationships between dimensionless width, depth and discharge during 1981 flood.

The bed load discharge rate is computed by the bed load formulae of Fukuoka as shown in equation (5), by Ashida & Michiue (1972) as shown in equation (7), and by Sato *et al.* (1958) as shown in equation (8).

$$\frac{q_{bk}}{\sqrt{sgd_k^{\ 3}}} = 17 P_k \tau_{*ek}^{\ \frac{3}{2}} \left(1 - \frac{\tau_{*ck}}{\tau_{*k}}\right)\left(1 - \frac{u_{*ck}}{u_*}\right) \tag{7}$$

$$\frac{q_{bk}}{\sqrt{sgd_k^{\ 3}}} = \varphi(n) P_k \tau_{*k}^{\ \frac{3}{2}} f\left(\frac{\tau_{ck}}{\tau_k}\right) \text{ with } \begin{cases} n \geq 0.025 : \varphi(n) = 0.623 \\ n < 0.025 : \varphi(n) = 0.623(40n)^{-3.5} \end{cases}, \ f\left(\frac{\tau_{ck}}{\tau_k}\right) = 1 - \frac{\tau_{*ck}}{\tau_{*k}} \tag{8}$$

where τ_{*k}, τ_{*ek} = dimensionless shear stress and effective shear stress for d_k, u_{*ck} = critical shear velocity for d_k, n = Manning's roughness coefficient.

Computational conditions

The initial bed forms before 1981 flood were surveyed in 1979 and are shown in Fig. 3. The boundary conditions of upstream and downstream ends are given by the observed water level hydrograph at the 15 km point and tide levels of the Otaru tidal observatory at sea 3 km from the river mouth, respectively. The Manning's roughness coefficients are determined so as to minimize the difference between the observed and the calculated water surface profiles at each time point.

Furthermore, we also compare the calculated bed form after the flood with the observed one, because the water surface profiles are affected by the bed variations during the flood. In this analysis, the validity of the roughness coefficients and bed variations are checked by the temporal changes in water surface profiles during the flood and the bed forms after the flood. As a result, n = 0.013 for the main channel downstream of 8 km, n = 0.021 for the main channel upstream of 8 km and n = 0.050 for the flood plain are given. This computational method has an advantage that calculated bed variation during the flood may be verified by temporal changes in observed water surface profiles.

Computational results

Figure 6 shows dimensionless bed load discharge rate calculated by the formula of Fukuoka (equation (5)), Ashida & Michiue (equation (7)) and Sato *et al.* (equation (8)). In the case of the Ishikari River mouth, Equation (5) and equation (8) calculate similar amount of bed load discharge rate, and equation (7) calculates more bed load discharge rate than the other equations. Bed load discharge rate calculated by equation (5) (for d_k arbitrary grain diameter) corresponds with equation (4) (for d_r representative grain diameter).

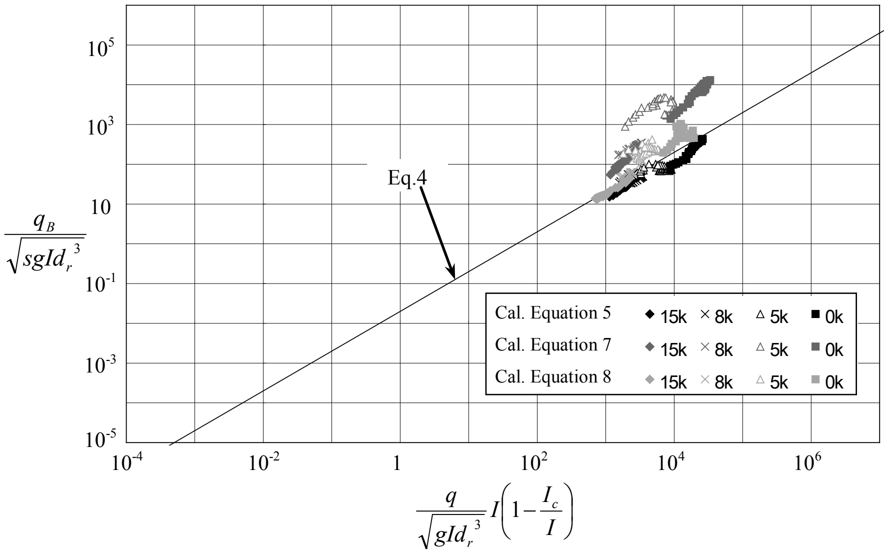

Fig. 6 Comparison of bed load discharge rate calculated by Equation 5, 7 and 8.

Figure 7 shows temporal changes in observed and calculated water surface profiles in the flood rising period. Figure 8 shows the comparison between observed and calculated longitudinal bed forms before and after the flood. There are small differences between observed water levels and calculated water levels of each calculation cases. In the Ishikari River mouth, suspended load causes the bed variations over a wide range and bed load causes local scours. The influences of local scours on the water surface profiles are small. Therefore, calculated water surface profiles of each calculation cases are similar. For the same reason, calculated average bed elevations of each calculation case are similar and coincide with observed average bed elevation, as shown in Fig. 8. However, clear differences between each calculation cases can be seen in the lowest bed elevations of the meandering section from 3 km to 8 km (Fig. 8). The scour depths of calculation using equation (7) are larger than those of the observation and the other calculation cases.

Figure 9 shows cross-sectional bed forms of observation and calculations in meandering section from 3.5 km to 6.5 km before and after the flood. The cross-sectional bed slopes of calculation using equation (7) are steeper than those of the observation after the flood. It is considered that the amount of lateral sediment discharge rate is calculated excessively. However,

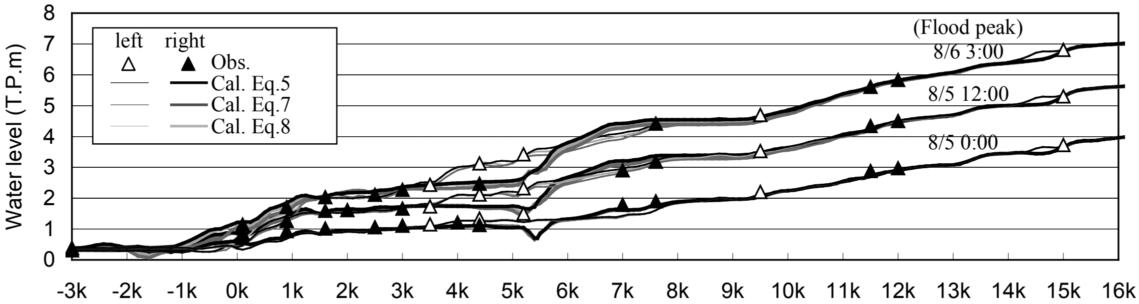

Fig. 7 Temporal changes in observed and calculated water surface profiles in flood rising period.

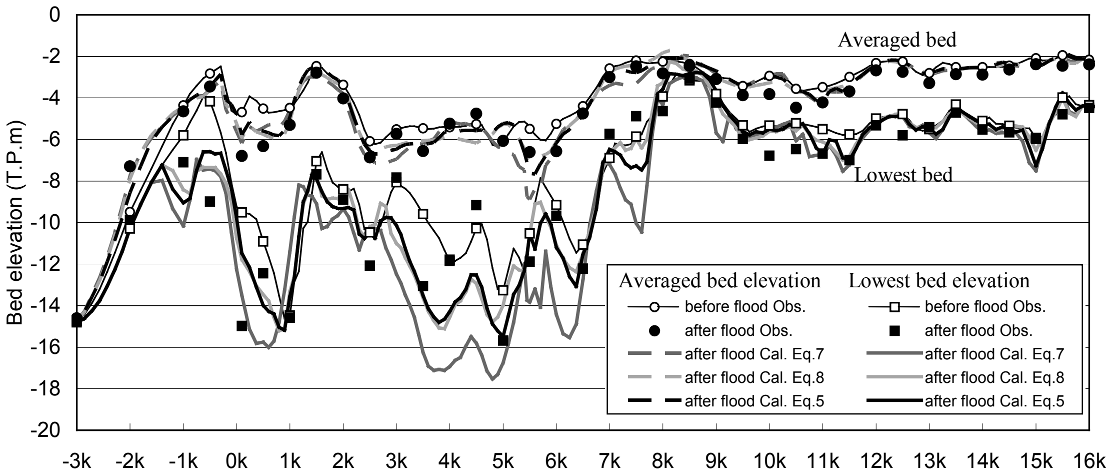

Fig. 8 Comparison between observed and calculated bed elevations before and after the 1981 flood.

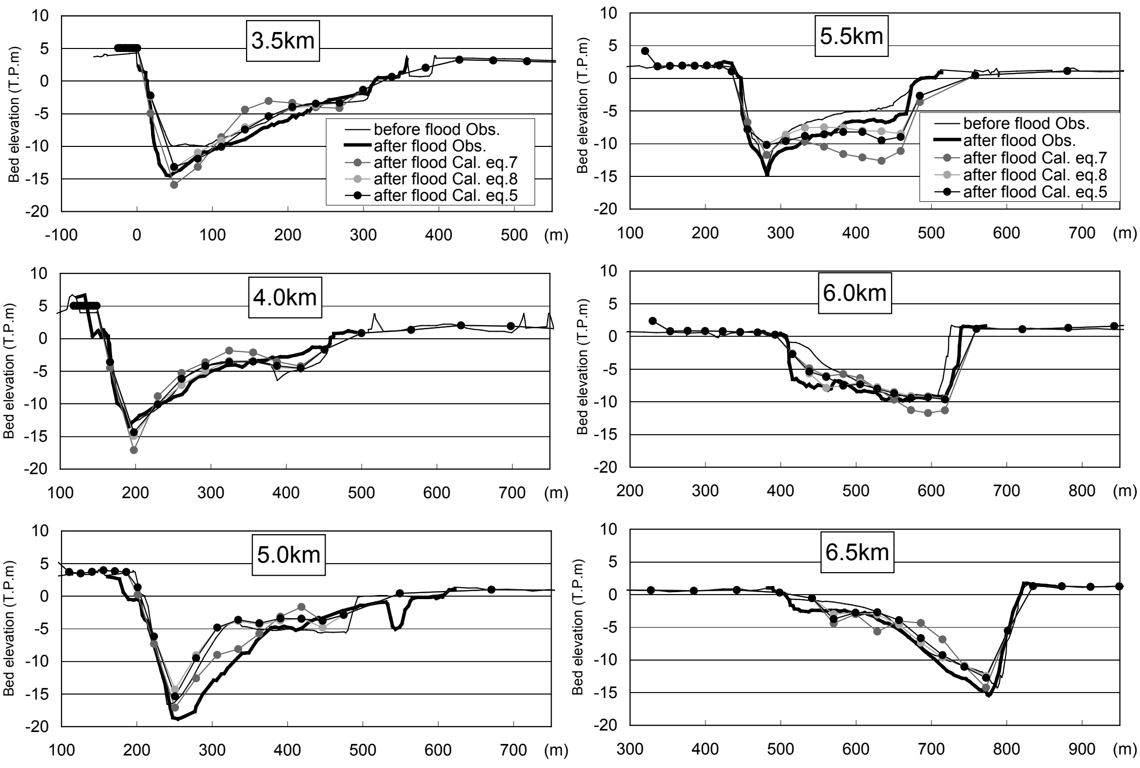

Fig. 9 Cross-sectional bed forms of observation and calculations in meandering section from 3.5 km to 6.5 km before and after the 1981 flood.

the cross-sectional bed slopes of calculations using equations (5) and (8) coincide with those of the observation. This suggests that equations (5) and (8) calculate the amounts of bed load discharge rates appropriately in the case of the Ishikari River mouth.

As shown in Fig. 4, the dimensionless bed load discharge rates measured in rivers varies widely even if the products of the dimensionless discharges and energy gradients are same. Therefore, Fukuoka's bed load formula will not always calculate bed load discharge rate appropriately. We have to examine for many rivers and floods which bed load formula evaluates the bed variations appropriately. Then, we should use the bed load formula which evaluates the bed variations appropriately, in each case.

CONCLUSION

In this paper, we develop the unsteady flood flow and bed variation analysis method with Fukuoka's bed load formula to calculate the 1981 flood of the Ishikari River mouth which caused a large bed scouring. We compare the amount of bed load discharge rates during the flood and the bed forms after the flood calculated by Fukuoka's bed load formula and the other formulae. Then, we investigate the effect of bed load formula on the accuracy of bed variation analysis.

In the case of the Ishikari River mouth, the bed load formula of Ashida and Michiue calculates more sediment discharge than the formulae of Fukuoka and Sato, Kikkawa and Ashida.

In the Ishikari River mouth, the wide range bed variation is mainly caused by suspended load, so the bed load has little effect on the average of bed variation. However, the local bed variations such as scours at meandering channel are affected by bed load.

The bed forms after the flood calculated by using the formulae of Fukuoka and Sato, Kikkawa and Ashida coincide with the observed bed forms. Especially, the cross-sectional bed forms after the flood in the meandering channel are well reproduced. This suggests that these formulae estimate the amounts of bed load discharge rates appropriately, in the case of the Ishikari River mouth.

Floods of many rivers for which bed load formula evaluates the bed variations properly need to be examined. The bed load formula which evaluates the bed variations should be used in each case.

REFERENCES

Ashida, K. & Michiue, M. (1972) Study on hydraulic resistance and bed-load transport rate in alluvial streams. *Proc. Japan Society of Civil Engineers* 206, 59–69.

Fukuoka, S. (2010) River engineering adaptations against the global warming – towards generalization of close-to-nature rivers, *Proceedings of the Japan Society of Civil Engineers*, *F JSCE* 66(4), 471–489.

Fukuoka, S., Watanabe, A. & Okada, S. (1998) Bed topography analysis in a compound meandering channel by using 3D numerical model with approximation of hydrostatics pressure. *Ann. J. Hydraul. Engng JSCE* 42, 1015–1020.

Inoue, T., Hamaki, M., Arai, N., Nakata, M., Takahashi, T., Hayashida, K. &Watanabe, Y. (2004) Quasi-three-dimensional calculation of riverbed deformation during 1981-year flood in the Ishikari River mouth. *Adv. River Engng JSCE* 10, 101–106.

Itakura, T. & Kishi, T. (1980) Open channel flow with suspended sediments. *J. Hydraul. Div.*, *Proc. ASCE* 106(HY.8), 1325–1343.

Mori, A. & Kishi, T. (1985) Calculation of three-dimensional meandering flow and bed topography. *Ann. J. Hydraul. Engng JSCE* 29, 691–696.

Nakato, T. & Kennedy, J. F. (1977) Field study of sediment transport characteristics of the Mississippi River near Fox Island (RM 355-6) and Buzzard Island (RM 349-50). *IIHR Report no. 201, Iowa Institute of Hydraulic Research.*

Nakato, T. & Vednal, J. L. (1977) Field study and test of several one-dimensional sediment-transport computer models for Pool 20, Mississippi River. *IIHR Report no. 237, Iowa Institute of Hydraulic Research*.

Okamura, S., Okabe, K. & Fukuoka, S. (2010) Numerical analysis of quasi-three-dimensional flow and bed variation based on temporal changes in water surface profile during 1981 flood of the Ishikari River mouth. *Adv. River Engng JSCE* 16, 125–130.

Sato, S., Kikkawa, H. & Ashida, K. (1958) Study on the bed-load transportation (1) *Reports for PWRI* 98, 13–30.

Shimizu, Y., Itakura, T., Kishi, T. & Kuroki, M. (1986) Bed variations during the 1981 August flood in the lower Ishikari River, *Ann. J. Hydraul. Engng JSCE* 30, 487–492.

Takagi, J., Makino, N., Takemoto, N. & Morita, Y. (1982) Field survey of flood flow and bed evolution in the lower Ishikari River. *Ann. J. Hydraul. Engng JSCE* 26, 57–62.

Uchida, T. & Fukuoka, S. (2009) A depth integrated model for 3D turbulence flows using shallow water equations and horizontal vorticity equations. In: *33rd IAHR Congress*, *Water Engineering for a Sustainable Environment*, 1428–1435.

Williams, G. P. & Rosgen, D. L. (1989) Measured total sediment loads (suspended loads and bed loads) for 93 United States streams. *US Geological Survey Open-File Report*, 89–67.