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Quasi-three dimensional computations for flows and bed variations in curved channel with gently sloped outer bank

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ABSTRACT: Secondary flows in curved channels cause bed scouring around outer banks. It is supposed that making slope of outer banks gentler is one of useful measures for mitigating scouring and bank erosion in curved and meandering channels. Therefore flow structures and bed variations around the gently sloped outer banks have to be understood to design slope of outer banks in actual rivers. However most of models proposed in curved channels were applied for the channels having a vertical wall. In this paper, applicability of the quasi-three-dimensional calculation model (the bottom velocity computation (BVC) method) is discussed for the experiments on flows and bed variations in curved channels with gently sloped outer banks (1V:2H and 1V:3H). From the comparison between calculation and experiment results, it is found that the BVC method reproduces vertical distributions of horizontal velocities and scouring depth around the gently sloped outer banks and provides a proper design of curved channels with gently sloped outer banks.

1 INTRODUCTION

Flow structures and riverbed topographies in curved and meandering reaches have been studied by many researchers. Secondary flows in curved reaches cause high velocities and bed scouring around outer banks and deposition at the inner banks. For preventing bed scouring and bank erosion around outer banks, various measures such as spur dikes, submerged vanes and footings have been implemented (e.g. Fukuoka & Watanabe 1993, Roca et al. 2007, Abad et al. 2008).

Making the slope of outer banks gentler is one of measures for outer bank protection (Fukuoka et al. 1995). Fukuoka et al. (1983) demonstrated that erodible-curved channels established stable cross-sections with gently sloped outer banks. Hence gently sloped outer bank is supposed to be a useful means for mitigating bed scours and bank erosions in curved and meandering reaches.

Flow structures and bed variations around the gently sloped outer banks have to be understood to design appropriate slope of outer banks in actual rivers. For the above reason, a numerical model which can estimates three-dimensional flows and bed variations around the gently sloped outer banks is required. However, most of the models proposed in curved or meandering channels (e.g. Engelund 1974, Nishimoto et al. 1992, Zeng et al. 2008) were applied for the channels having a vertical wall.

Otsuki et al. (2001) conducted experiments and numerical simulations for a curved channel with sloped outer banks in order to estimate forces acting on the side slope for designing bank revetments. Maynord (1992) carried out experiments to study riprap stability by using a straight-meandering channel with gently sloped banks (1V:2H and 1V:3H). Finnie et al. (1999) developed the calculation method based on a depth-integrated model that takes into account development and decline of secondary flows and applied the model to Maynord's (1992) experiments.

3D turbulence models were adopted for flows in curved or meandering flumes with gently sloped banks (Ye & McCorquodale 1998, Wang et al. 2009). 3D turbulence models are useful to investigate three-dimensional flow structures in channels. However, it takes much computational time for applying to actual rivers.

Fukuoka et al. (1995) conducted research on the effects of slope of outer banks on flow structures and bed variation for a uniformly curved laboratory channel by the quasi-three dimensional model (Fukuoka et al. 1992). Their model is useful for design of equilibrium cross-sectional form in curved channel, but not of non-equilibrium changes in flow structures and bed of channel with gently sloped outer banks in actual rivers.

Fukuoka & Uchida (2011, 2013) have developed a quasi-three dimensional model which can estimate the bottom velocities and three-dimensional velocities due to depth-averaged transport and deformation of vorticities (the Bottom Velocity Computation (BVC) method).

In this paper, applicability of the BVC method is examined to Fukuoka et al.'s (1995) flows and bed variations experiments in the uniformly curved channel with gently sloped outer banks.

2 NUMERICAL ANAYSIS METHOD

2.1 *The bottom velocity computation (BVC) method*

The BVC method is a practical numerical model which can calculate bottom velocities and vertical distributions of horizontal velocities in the framework of the two-dimensional approach. Since pressure distributions in uniformly curved channels are hydrostatic except near the wall, hydrostatic distributions of the BVC method is applied in the present analysis although the BVC methods are used for both flows with hydrostatic and non-hydrostatic pressure distributions (Uchida & Fukuoka 2011).

The water depth and depth-averaged horizontal velocities are calculated by depth-integrated continuity equation (Equation 1) and depth-integrated horizontal momentum equations considering momentum transport due to vertical distributions of horizontal velocities (Equation 2).

$$\frac{\partial h}{\partial t}+\frac{\partial U_j h}{\partial x_j}=0 \tag{1}$$

$$\frac{\partial U_i h}{\partial t}+\frac{\partial U_i U_j h}{\partial x_j}=-gh\frac{\partial z_s}{\partial x_i}-\frac{\tau_{bi}}{\rho}+\frac{1}{\rho}\frac{\partial h(\tau_{ij}-\rho\overline{u'_i u'_j})}{\partial x_j} \tag{2}$$

where h = water depth; U_i = depth-averaged horizontal velocities; g = gravitational acceleration; z_s = water level; τ_{bi} = bed shear stress; τ_{ij} = horizontal shear stress due to turbulence; and u'_i, u'_j = deviation velocity components from depth-averaged velocity.

τ_{bi} and τ_{ij} are described as follows, respectively.

$$\tau_{bi}=\rho {c_b}^2 u_{bi}|u_b| \quad , \quad u_b=\sqrt{u_{bx}^2+u_{by}^2}$$
$$\frac{1}{c_b}=Ar+\frac{1}{\kappa}\ln\left(\frac{\delta z_b+\delta z_0}{k_s}\right) \tag{3}$$

$$\tau_{ij}=2\nu_t\left(\frac{\partial U_i}{\partial x_j}-\frac{\partial U_j}{\partial x_i}\right) \tag{4}$$

where, u_{bi} = bottom velocities; $\delta z_b=(e^3-1)/h$; δz_0: the origin height of log law; k_s = equivalent roughness height; and ν_t = eddy viscosity coefficient. ν_t is calculated from a local equilibrium model that takes into account the vertical distributions of horizontal velocities.

One of the characteristics of the BVC method is to calculate bottom velocities which are important for bed variation analysis. The bottom velocities are obtained by the Equation 5, which is derived by depth-integrated horizontal vorticity.

$$u_{bi}=u_{si}-\varepsilon_{ij3}\Omega_j h \tag{5}$$

where u_{si} = horizontal velocities at water surface; ε_{ij} = Levi-Civita symbol; and Ω_i = depth average of horizontal vorticity.

The BVC method is composed of depth integrated horizontal vorticity equation (Equation 6), water surface velocity equation (Equation 7) in addition to depth-integrated continuity equations (Equation 1) and depth-integrated horizontal momentum equations (Equation 2).

$$\frac{\partial \Omega_i h}{\partial t}=ER_{\sigma i}+P_{\omega i}+\frac{\partial hD_{\omega ij}}{\partial x_j} \tag{6}$$

where ER = rotational term of vertical distribution of vorticity; P_ω = production term of vorticity from bottom thin vortex layer; and $D_{\omega ij}$ = horizontal vorticity flux due to convection, rotation, dispersion and turbulence diffusion.

$$\frac{\partial u_{si}}{\partial t}+u_{sj}\frac{\partial u_{si}}{\partial x_j}=-g\frac{\partial z_s}{\partial x_i}+P_{si} \tag{7}$$

where P_{si} = production term of surface velocities (shear stress under the water surface layer).

The vertical distributions of horizontal velocities are described by the cubic functional form (Equation 8) using depth-averaged horizontal velocities U_i, horizontal velocities at water surface u_{si}, and bottom velocities u_{bi} with the assumption that vertical gradient of velocity at water surface is zero.

$$u'_i=u_i-U_i=\Delta u_i(12\eta^3-12\eta^2+1)-\delta u_i(4\eta^3-3\eta^2) \tag{8}$$

where $\Delta u_i=u_{si}-U_i$; $\delta=u_{si}-u_{bi}$; and $\eta=(z_s-z)/h$. The momentum transport due to the vertical distribution of horizontal velocities in Equation 2 is calculated by integrated Equation 8 respect to depth. The vertical distributions of vorticity are defined as the differential form of Equation 8.

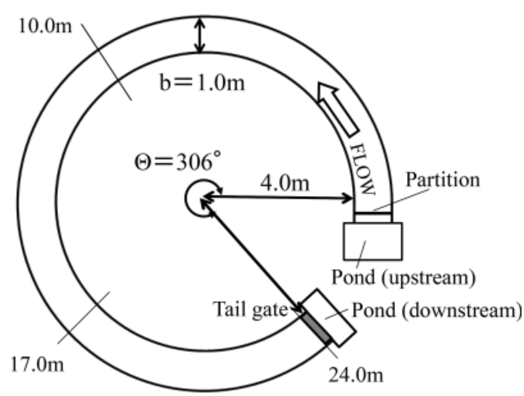

Figure 1. Plan form of the flume.

2.2 *Bed variation computation method*

Experiments (Fukuoka et al. 1995) were conducted under the condition of no suspended sediments. The bed variation computation took only bed load into account. The temporal variation in bed elevations are evaluated by the two-dimensional continuity equation for sediment. The bed load transports rate is calculated by the Ashida & Michiue bed load formula (Ashida & Michiue 1972). The tractive force is evaluated by bottom velocities solved from Equation (5). The critical tractive force is calculated by Iwagaki formula (τ_{*c0} = non-dimensional critical tractive force is 0.034 for d = 0.8 mm of the experiments). Effect of bed slope on the tractive forces and critical tractive forces of sediment particle are considered by Fukuoka & Yamasaka equations (Fukuoka & Yamasaka 1983).

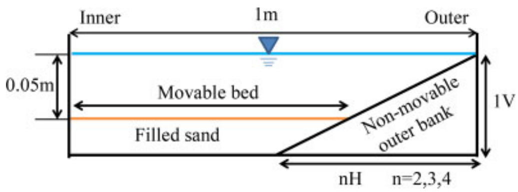

Figure 2. Initial cross-sectional form of flume.

Table 1. Experimental conditions (Fukuoka et al. 1995).

Channel length	24 m
Channel width	1 m
Water discharge	1.8×10^{-2} m^3/s
Sediment discharge	1.7×10^{-6} m^3/s
Sediment diameter	0.8×10^{-3} m
Bed slope	1/500
Central radius of curveture	4.5 m

3 ANALYSIS TO FLOWS AND BED VARIATIONS IN A CURVED CHANNEL WITH GENTLY SLOPED OUTER BANKS

3.1 *Experimental and computational conditions*

Fukuoka et al. (1995) investigated flow structures and bed variations for a uniformly curved channel with gently sloped outer banks as shown in Figure 1 and 2. The experimental conditions are summarized in Table 1. The sand was filled and leveled as shown in Figure 2. The steady discharge and sediment supply were given at the upstream end of the channel. After the equilibrium condition of the flow was established, longitudinal water level, bed topographies and vertical distributions of streamwise velocities and secondary flow velocities were measured. The experiments were conducted in four cases where the outer

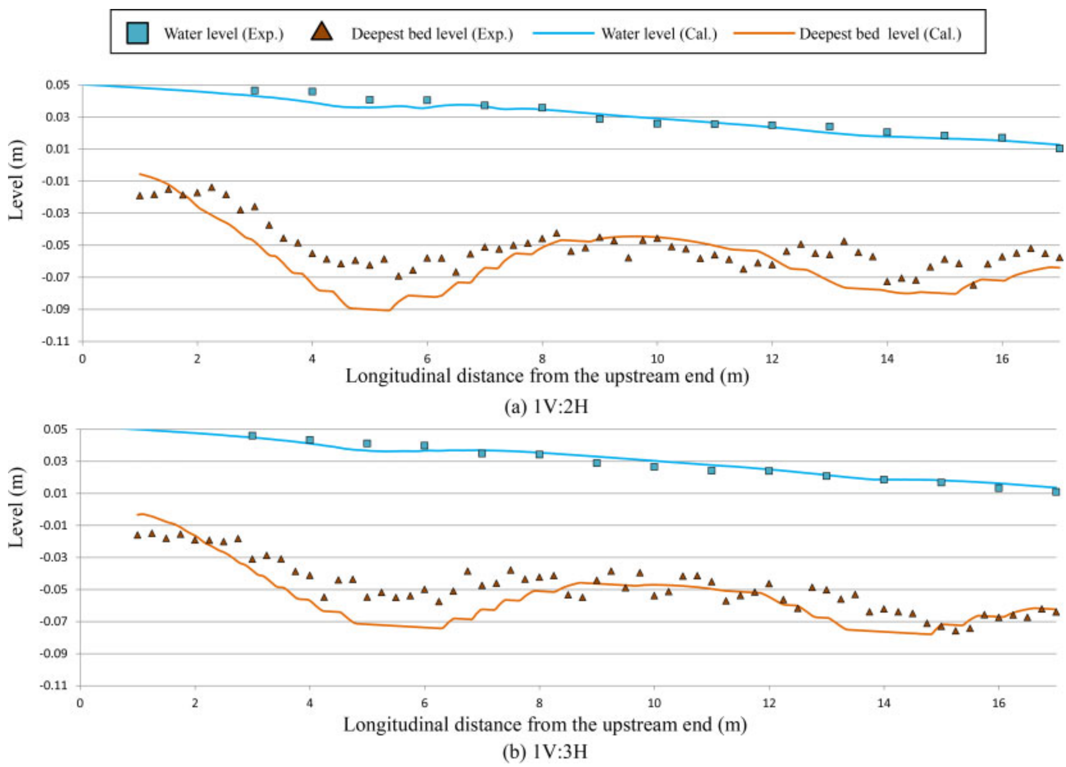

Figure 3. Comparisons of longitudinal water surface profiles and deepest level of channels bed between the experiments (Fukuoka et al. 1995) and the calculation results.

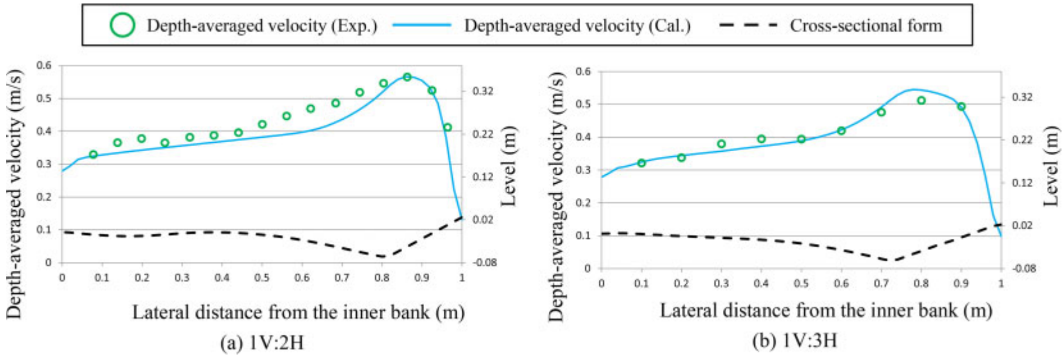

Figure 4. Comparisons of the depth-averaged velocity distributions along the cross-section between the experiments and calculations.

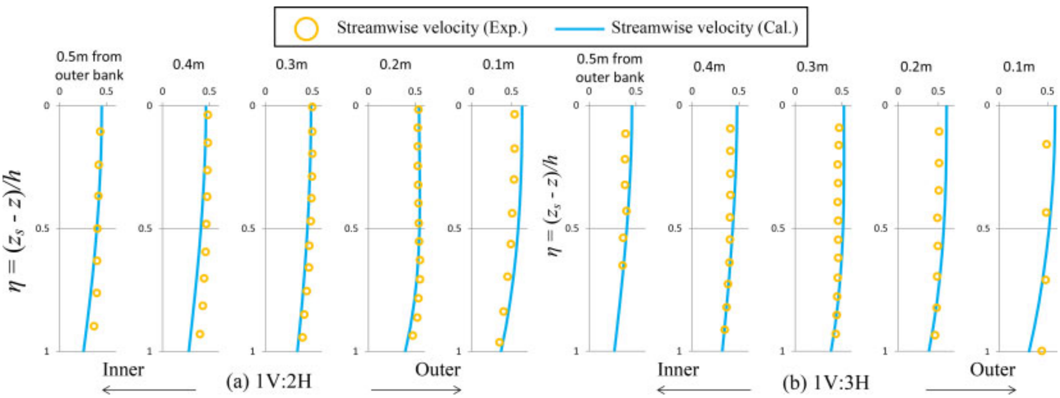

Figure 5. Comparisons of vertical distributions of the streamwise velocity (m/s) between the experiments and calculations. ($\eta = 0$: Water surface, $\eta = 1$: Bed surface).

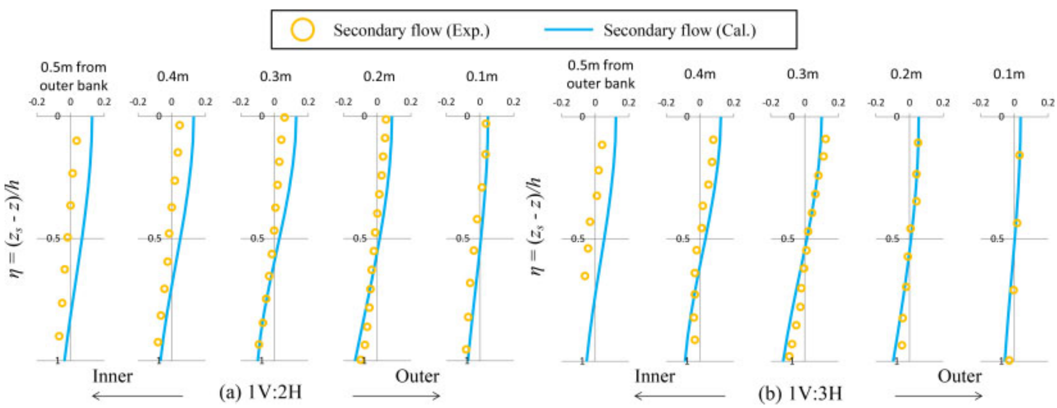

Figure 6. Comparisons of vertical distributions of the secondary flow/cross sectional average velocity between the experiments and calculations. ($\eta = 0$: Water surface, $\eta = 1$: Bed surface).

banks have the vertical wall and slope of 1V:2H, 1V:3H and 1V:4H. BVC method is applied to the case of 1V:2H and 1V:3H experiments.

Steady discharge ($1.8 \times 10^{-2}\, m^3/s$) and sediment discharge ($1.7 \times 10^{-6}\, m^3/s$) are supplied at the upstream end of the channel in the experiments and calculations. The bed material is a uniform sand of $d = 0.8\, mm$. The calculation grid size is 0.035 m in the longitudinal direction and 0.020 m in the lateral direction. The equivalent roughness height k_s is given by $3d$ so as to agree with longitudinal water surface profiles of the experiments.

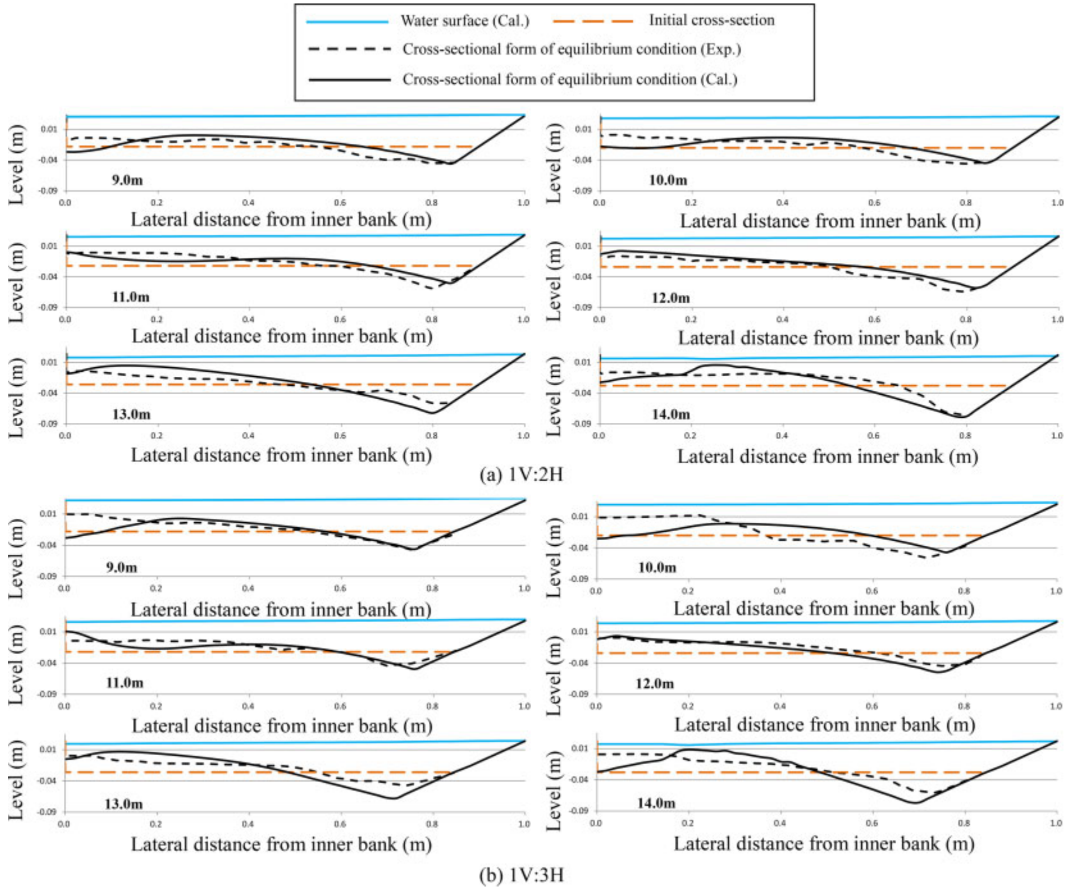

Figure 7. Comparisons of cross-sectional form between measurements and calculation results.

3.2 *Calculation results*

The measurement (Fukuoka et al. 1995) and calculation results of longitudinal water level and deepest bed level are compared in Figure 3. The calculation results of the 1V:2H and 1V:3H experiments explain well longitudinal water level profiles and deepest bed levels in the reach from 11 m to 13 m where velocity measurements were conducted. On the other hand, calculated bed levels in the upstream reach are deeper than measured ones and calculated water levels are slightly lower than the measured data. This reason would be considered as follows: A partition is installed at the upstream end of the channel to prevent the sand to be washed away by the discharge inflowing from the upstream pond. However, it made flows and sediments transports near the upstream end disturb although the channel cross-section is uniform. The partition was not considered in the present analysis.

Figure 4 shows measured and calculated distributions of the depth-averaged streamwise velocities in the cross-sections. They are average values of velocities measured at 11.0 m, 12.0 m, 12.5 m and 13.0 m cross-sections. Vertical distributions of streamwise velocities and secondary flows are also averaged in a similar manner. In the 1V:2H experiment (Figure 4(a)), a high-velocity zone is seen near the outer bank. On the other hand, the high-velocity zone in the 1V:3H experiment (Figure 4(b)) shifted toward the channel center compared with the 1V:2H experiments. Moreover, maximum velocity in the 1V:3H experiment become smaller than that of the 1V:2H experiment. It shows that gentler slope of outer banks makes velocity distributions uniform and decreases velocities near the outer bank. The calculation results explain well the characteristics of velocity distributions in the 1V:2H and 1V:3H experiments.

Figure 5 compares the results of experimental and calculated vertical distributions of streamwise velocities (0.1 m, 0.2 m, 0.3 m, 0.4 m and 0.5 m from the outer bank). Vertical distributions of streamwise velocities become uniform near outer banks because large momentum at the water surface is transported to the bottom by the secondary flows. Calculation results almost agree with experimental results.

Figure 6 shows comparisons of secondary flow velocities between the experiments and calculations. The secondary flow velocities are shown in dimensionless form by cross-sectional averaged-velocities.

Calculation results reproduce measured data around the outer bank. On the other hand, calculation results in the inner bank are in rather poor agreement with measured data compared near outer bank. It is caused by formation of sand bars around the inner banks which were not seen in experiments (to be discussed later), because secondary flow velocities calculated in the fixed bed condition made by measured bed topographies almost agreed with measured secondary currents in the inner bank (Sasaki et al. 2015).

Figure 7 shows measured and calculated equilibrium cross-sections at 9.0 m, 10.0 m, 11.0 m, 12.0 m, 13.0 m and 14.0 m. Brown lines in Figure 7 are initial cross-sections at each section. As shown by black broken lines of Figure 7 (a) and (b), scouring positions in the 1V:3H experiment moved towards the center of the channel and scouring depth decreased compared to the 1V:2H experiment. Calculation results shown by black solid lines almost reproduced scouring positions and depth around the gently sloped-outer banks. However, sand bars which had not be seen in the experiment were formed around the inner bank in the both 1V:2H and 1V:3H calculations. It can be presumed that the sand bars were generated by large sediment transports due to the excessive scouring around the upstream reach (see Figure 3 (a) and (b)) in the calculation.

These results show that the BVC method can calculate flows and bed variations in curved channels with gently sloped-outer banks.

4 CONCLUSIONS

The bottom velocity computation method (BVC method) was applied to Fukuoka et al.'s (1995) experiments. The main conclusions derived from this study are shown below.

1. The BVC method reproduces vertical distributions of streamwise velocities and secondary flow velocities in the uniformly curved channels with gently sloped-outer banks (1V:2H and 1V:3H).
2. The BVC method can explain scouring positions and depth around the gently sloped-outer banks.
3. Above results suggest that the BVC method might be applied for designing actual rivers having gently sloped-outer banks.

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