

# Quasi-3D two-phase modelling of fluid and sediment dynamics based on a non-hydrostatic depth integrated model with a dynamic rough wall law

**Tatsuhiko Uchida**

Research and Development Initiative, Chuo University  
1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan  
Tel. +81-3-3817-1617

E-mail: utida@tamacc.chuo-u.ac.jp

**Shoji Fukuoka**

Research and Development Initiative, Chuo University  
1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan  
Tel. +81-3-3817-1625

E-mail: sfuku@tamacc.chuo-u.ac.jp

*The assumption of the equilibrium flow in the vicinity of the bed is questionable to calculate sediment transport for complex flows, such as dam-break flows and flows around structures. This study proposed a new two-phase depth-integrated model for the applications of large scale sediment transport phenomena and bed morphology in geophysical flows. The model for fluid phase is based on the non-hydrostatic quasi-3D method with employing a dynamic rough wall law.*

*It was confirmed through the comparisons with previous formulae of bed load and suspended load that the present model provided similar results for equilibrium conditions. Then, the present model was applied to the experiment on dam-break flow on movable bed. The comparisons between the experimental results and results calculated by the present model and the previous models demonstrate the validity of the present model and advantages of evaluating vertical velocity distributions and introducing two-phase model.*

**Key Words:** *two-phase model, dynamic wall law, depth integrated model, bottom velocity computation method, vertical velocity distribution, dam-break flow*

## 1. Introduction

For the conventional bed variation analysis method, the bed shear stress, which is used as the bed tractive force acting on sediment particles to calculate sediment transport rate, has been evaluated with an equilibrium wall law (EWL) and the assumption of the equilibrium flow condition near the bed. However, for the case of complex flows, such as dam-break flows and flows around hydraulic structures, the assumption of the equilibrium flow is not suitable even for the flow in the vicinity of the bed which includes roughness layer under the bed surface [1], [2]. As long as EWL is employed, non-equilibrium motions of sediment particles and dynamic interactions between fluid and sediment motions cannot be evaluated correctly.

Because it is difficult or even impossible to fully understand fluid-sediment dynamics and interactions only by laboratory experiments, a high or fully resolution computational fluid dynamic model coupled with sediment particles equations has been developed [3]. Although the numerical experiments with these models have a great role for investigating particle-scale dynamics between fluid flow and sediment particle motions, it can be more attractive for practical applications in large domains to develop a reliable depth-integrated model. The quasi-3D models with the ability of evaluating variations in vertical distributions of velocity have been developed for bed variation analysis in curved or meandering channels [4]. Some two-dimensional two-phase models are proposed for dam-break flow over mobile bed [5]. However, to our knowledge, no quasi-3D two-phase models, which can evaluate

both 3D flow structure and momentum transfer between fluid and sediment motions with the framework of 2D model, have been available so far.

In this study, a new two-phase depth-integrated model is derived for the calculation of large scale sediment transport phenomena and bed morphology in geophysical flows. The validity of the present model and advantages of evaluating vertical velocity distributions and introducing two-phase model are discussed though the comparisons with previous experimental results for dam-break flow on mobile bed [6] and existing depth integrated model with conventional single phase sediment transport models [4].

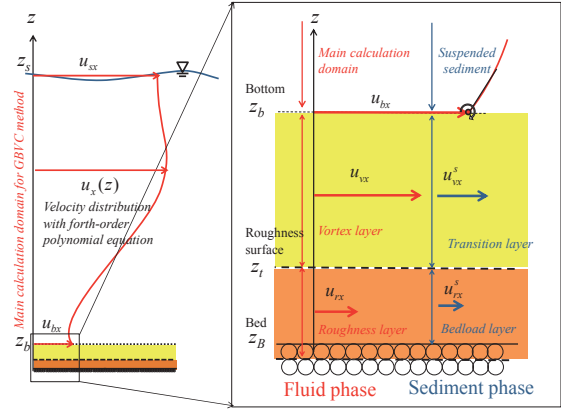
## 2. Quasi-3D two-phase model

### 2.1. Fluid phase

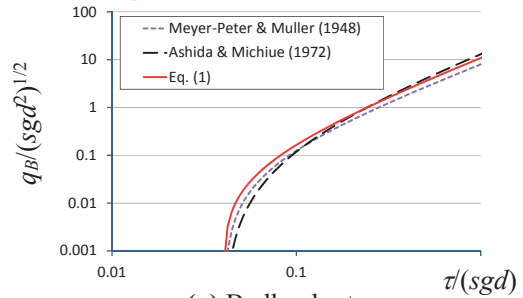
The calculation method for fluid phase is based on the General Bottom Velocity Computation method with employing Dynamic rough Wall Law (GBVC4-DWL [2]), taking into account interactions with sediment phase. The calculation domain is divided into three layers in vertical directions: main calculation domain, vortex layer and roughness layer, as shown in Fig.1. For the vertical distribution of horizontal velocity, the fourth-degree polynomial velocity distribution is adopted with the constrain conditions of depth averaged velocity, water surface and bottom velocity, and the vertical gradient of velocity at water surface and bottom. These quantities to evaluate vertical distributions of velocity and pressure in the main calculation domain are calculated with several depth-integrated equations which are derived from Reynold's averaged continuity, momentum and vorticity equations. The bottom boundary conditions of the main calculation domain for depth integrated momentum and vorticity equations are given by the DWL with the continuity and momentum equations for the vortex and roughness layers, instead of the uniform velocity distribution under the bottom with the EWL. The avoidance of assuming the equilibrium flow condition near the bed, in which sediment particles are driven actively by fluid flow, is a significant advantage in the present method to take into account interactions between fluid and sediment motions: two-phase modelling. The detail of the GBVC4-DWL method may be found in the previous literature [2].

### 2.2. Sediment phase

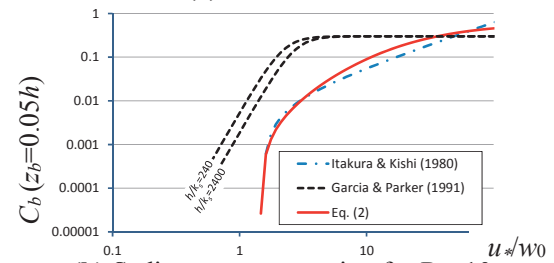
The model for sediment phase newly introduced in this study has a different sub-model developed for each layer. The bedload layer is assumed to be in the roughness layer. The particle skeleton stress is considered to be significant for the bed load layer with dense sediment concentration. A two-phase model, which consists of continuity and momentum equations for both fluid and sediment phases with the variation in the thickness of the layer under the assumption of the constant sediment concentration



**Fig.1** Computational domain of the present quasi-3D two-phase model



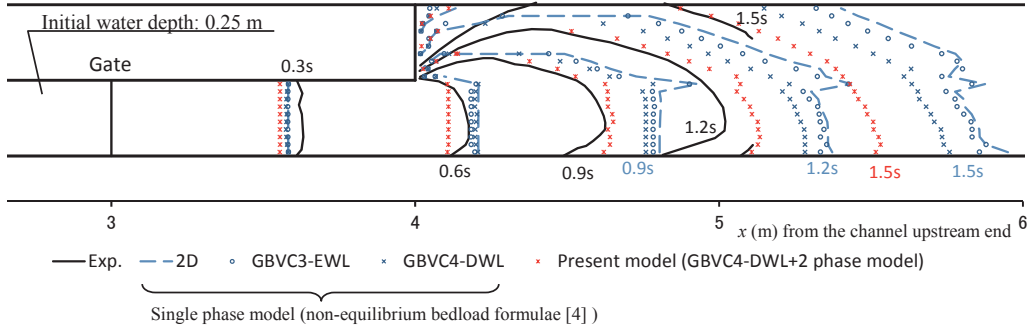
**(a)** Bedload rate



**(b)** Sediment concentration for  $R_p=10$

$R_p = (sgd^3/\nu^2)^{1/2}$ ,  $\nu$ =kinematic viscosity,  $h$ =water depth,  $k_s$ =equivalent roughness height

**Fig.2** Comparisons with previous formulae for equilibrium conditions



**Fig.3** Water front propagations in the experiment [6] and several computation methods

[5], is applied to the bedload layer, considering interactions with the upper transition layer. The existing formulae are adopted for evaluating bed load entrainment rate  $E$  and deposition rate  $D$ , along the line of Greco et al. [5]. The hydrodynamic force acting on bedload sediment is calculated with roughness and vortex layers velocity without EWL which has been employed for previous sediment transport models [4],[5]. Over the bedload layer, we consider the transition layer in the vortex layer, in which a dilute two-phase model without particle-collision stress is introduced, calculating sediment concentration and velocity. In the main computational domain, a single-phase model is adopted with a depth integrated advection-diffusion equation of sediment concentration. The interactions between the layers are dynamically evaluated without assuming equilibrium flow conditions. For equilibrium flow condition, the governing equations of the present model for sediment phase yield following bedload rate  $q_B$  (1) and sediment concentration at the bottom  $C_b$  ( $z_b=0.05$ ) (2):

$$\frac{q_B}{\sqrt{sgd}} = c_L \left( \frac{c_p}{c_d} \right)^{2/3} (\tau_* - \tau_{*c}) \left( \sqrt{\tau_*} - \sqrt{3C_D c_L^2} \right) \quad (1), \quad C_b = \frac{C_B}{1 + w_0 / \alpha u_*} \frac{1 - a(w_0 / \alpha u_*)}{1 + (1 - a)(w_0 / \alpha u_*)} \quad (2)$$

where,  $s$ =submerged specific gravity of sediment,  $g$ =acceleration of gravity,  $d$ =sediment particle diameter,  $c_L=14$ ,  $c_p=0.02$ ,  $c_d=0.025$ ,  $\tau_* = u_*^2 / sgd$ ,  $u_*$ =shear velocity,  $\tau_{*c}$ =dimensionless critical Shields stress,  $\mu_d$ =dimensionless dynamic friction coefficient ( $\mu_d=0.6$ ),  $C_D$ =drag coefficient ( $C_D=0.4$ ),  $C_B$ =sediment concentration at the bed ( $C_B=0.6$ ),  $w_0$ =settling velocity,  $\alpha = \kappa/6$ ,  $a=0.1$ . It is confirmed from Fig. 2 that the bed load rate and sediment concentration calculated by present model for equilibrium flow condition are within the range of those estimated by previous formulae [7].

### 3. Model application to dam-break flow over mobile bed

The present model (GBVC4-DWL for two phase model) is applied to the experiment on dam-break flow on movable bed with a left-side sudden enlargement [6]. The experiment was conducted in the flat bed channel with 6 m long, 0.25 m to 0.50 m width, a gate 1m upstream from the enlargement section, as shown in Fig. 3. The uniform sand with  $d_{50}=1.72$  mm was used. Refer to the literature [6] for details. For the calculations, the above experimental conditions are given with equivalent roughness  $k_s=d_{50}$  and the depth of the log law origin  $dz_0=0.3d_{50}$  for evaluating bed roughness. Numerical calculations with 2D, GBVC3-EWL and GBVC4-DWL coupled with the previous single phase non-equilibrium bed load model [4] are conducted for the same condition and parameters used in the present model.

Fig.3 shows comparisons of water front propagation between experimental results [6] and several computation results. The water front for 0.3 sec. is considered to be influenced by the gate opening operations and initial complex sediment motion near the gate. The water fronts after 0.6 sec. computed with the single phase model are propagated considerably far from that of experiment. The effect of sophistication of fluid phase model for the main computational domain on the water front propagation is relatively small (2D vs GBVC3-EWL). The BVC4-DWL with single phase model still underestimates movable bed resistance. On the other hand, the present model provides good agreement with experimental results. The above indicates that the non-equilibrium flow and momentum exchange between fluid and sediment phase near the bed are essential to evaluate the resistance of movable bed for dam-break flows. Fig.4 shows bed topography after dam break flow by experiment [6] and several

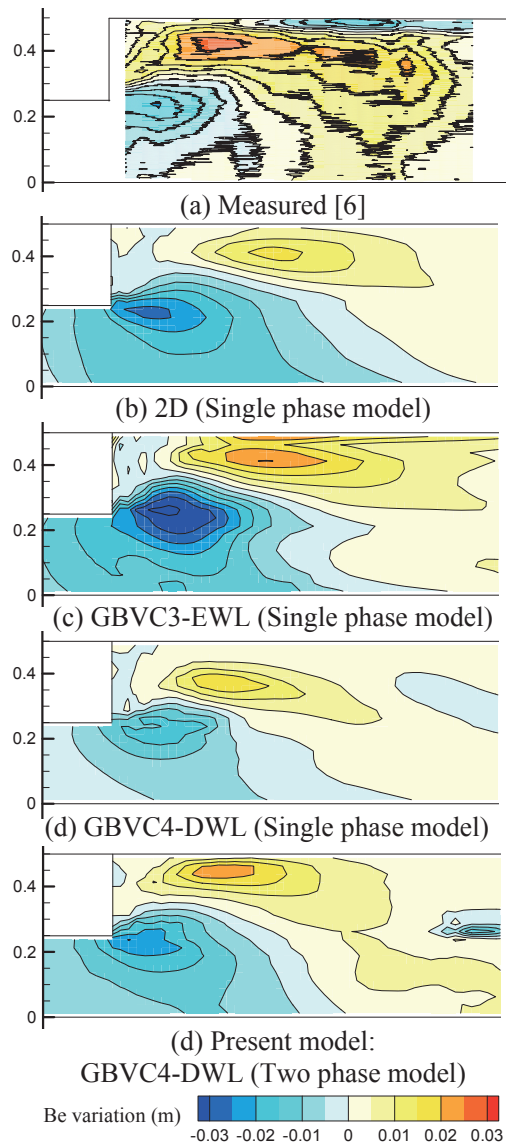
computation methods. For the calculation with single phase model (b)-(c), sediment deposition areas occur downstream compared to experiment. The calculation results in Fig.4 show the effect of fluid phase models on the calculation of bed topography. However, it should be noted from the comparison with measured result that the sophistication of the fluid phase model does not necessarily improve the bed variation calculation unless an adequate sediment phase model is employed. It is investigated in the preliminary calculations that the bed variation calculation results by BVC4-DWL with single phase model depend on the definition of the bed tractive force for computing bedload rate. The calculation result by the present model explains well about experimental bed topography except for the deposition height and local scour along the left bank investigated for the experiment.

#### 4. Conclusions

A new two-phase depth-integrated model based on non-hydrostatic quasi-3D flow model with a dynamic wall law (GBVC4-DWL) is proposed for the applications of large scale sediment transport phenomena and bed morphology in geophysical flows. The comparisons for dam-break flow over movable bed between the experimental results and results calculated by the present model and the previous models demonstrate the validity of the present model and advantages of evaluating vertical velocity distributions and introducing two-phase model.

#### References

- [1] Uchida, T. and Fukuoka, S. (2015) A new calculation method for local three dimensional flows by using the non-hydrostatic depth integrated model (BVC method) with dynamic wall-law for rough bed, *Journal of Japan Society of Civil Engineers, Ser. B1 (Hydraulic Engineering)*, Vol. 71, No. 2, pp.43-62, 2015, in Japanese.
- [2] Uchida, T., Fukuoka, S., Papanicolaou, T. and Takiris, A.G. (2016): Non-hydrostatic quasi-3D model coupled with dynamic rough wall law for simulating flow over rough bed with submerged boulders, *Journal of Hydraulic Engineering*, accepted.
- [3] Kidanemariam, A. G. and Uhlmann, M. (2014) Interface-resolved direct numerical simulation of the erosion of a sediment bed sheared by laminar channel flow, *International Journal of Multiphase Flow*, Vol.67, 174-188.
- [4] Uchida, T. and Fukuoka, S. (2014) Numerical calculation for bed variation in compound-meandering channel using depth integrated model without assumption of shallow water flow, *Advances in Water Resources*, Vol.72, pp.45-56, 2014.
- [5] Greco, M., Iervolino, M., Leopardi, A., and Vacca, A. (2012) A twophase model for fast geomorphic shallow flows." *Int. J. Sediment. Res.*, 27(4), 409-425.
- [6] Goutiere, L., Soares-Frazaõ, S. and Zech, Y. (2011) Dam-break flow on mobile bed in abruptly widening channel: experimental data, *Journal of Hydraulic Research*, 49:3, 367-371.
- [7] Garcia, M. H. (2008) *Sedimentation engineering: Processes, management, modeling and practice*, ASCE, Reston, VA..



**Fig.4** Comparisons of bed topography caused by dam break flow